Schooled by Trade? Retraining and Import Competition *

Lucas Conwell † Trevor C. Williams ‡

January 26, 2022

Abstract

Retraining is a policy tool that can help workers displaced by import competition, but there is surprisingly little work in economics on retraining and trade adjustment. Using administrative data from Germany, a highly open economy with extensive government-subsidized retraining programs, we provide evidence that workers retrain in response to import competition. To understand how retraining changes the gains from trade, we present a search model in which heterogeneous workers may choose to retrain while unemployed. In our empirical calibration, retraining increases the gains from trade by 7% in the aggregate. Some worker groups gain five times as much, while others gain virtually nothing.

1 Introduction

Trade creates winners and losers. A central question for trade policy is how to balance these competing interests. Retraining is one such policy tool that can help workers who lose their jobs due to foreign competition. Worker retraining or "workforce development" inevitably accompanies any discussion of trade and its impacts on workers.[1] In the US, the budget passed by the House of Representatives in November 2021 devoted

*Preliminary and incomplete. We thank Costas Arkolakis, Lorenzo Caliendo, Giuseppe Moscarini, Michael Peters, and seminar participants at Yale for the helpful comments and suggestion. We thank the research staff at the FDZ in Nuremberg for their kind assistance with the data. We gratefully acknowledge funding from the MacMillan Center at Yale.

†Yale University. lucas.conwell@yale.edu

‡Yale University. trevor.c.williams@yale.edu

[1] See, for example, Alden (2016) in PBS or Morath (2018) in *The Wall Street Journal*.

$24 billion in funding to "worker training, employment services, re-entry programs, apprenticeships, industry partnerships and other programs" and an extra $6 billion in trade adjustment assistance (TAA) training grants (Parlapiano and Bui 2018). A large body of work in economics has documented the labor market effects of trade liberalization, ranging from changes in employment and wages to occupation-switching, sectoral reallocation, and migration. Researchers have also estimated the effects of retraining programs on individual workers. However, prior literature has not connected trade and retraining, empirically or theoretically. In this paper, we document that import competition significantly increases retraining take-up and propose a framework to clarify how retraining changes the gains from trade.

Retraining is a key component of active labor market policy in Germany, with the government spending about 0.04% of GDP on retraining subsidies (Bundesagentur für Arbeit 2021d). Our study focuses on short-term retraining programs for the unemployed. These programs typically last a few weeks, and so are a flexible, low-cost alternative to traditional education. Retraining is designed to smooth the transition into employment by helping workers with job search and skill acquisition. Some programs cover basic, transferable skills, while others target specific fields or occupations. We start by documenting a new set of facts. Around a fifth of workers have ever participated in retraining; programs are brief, with a median duration of four weeks; and about three quarters of retraining occurs out of unemployment. Given the last fact, we choose to focus our attention on retraining by the unemployed.

Using German administrative data, we first show that import-competing workers retrain at elevated rates. After retraining, they earn higher wages and re-enter employment more quickly. Retraining is positively and significantly correlated with subsequent sector switching, consistent with a trade adjustment story.

Next, we present a tractable search model of retraining. When workers can retrain and change their human capital, labor supply responds endogenously—but with a lag—to wage changes. The response is slow if frictions to retraining are high. The model provides closed forms for the retraining rate, the speed of adjustment to shocks, and the aggregate gains from trade in the presence of retraining. Retraining adds a new term to the standard expression for the gains from trade. It reflects option value of upgrading human capital and is increasing in the degree of mismatch between the previous skill distribution and the optimal skill distribution under the new price regime.

The key parameters of the model govern retraining benefits, in terms of higher job finding rates and productivity, and costs, in terms of a sunk utility cost. We estimate the model on the microdata using a two-stage procedure that draws on the expectation maximization algorithm and indirect inference. Finally, we evaluate retraining by conducting two counterfactual experiments: first, in our "gains from trade" counterfactual, we compare the change in welfare from a decrease in manufacturing world prices with and without the possibility of worker retraining. The gains from trade are about 7% higher with retraining, reflecting workers' ability to direct their training towards newly valuable skills. Retraining has substantial distributional effects: some workers' gains are almost 40% higher, while others' are actually lower. Second, in our "retraining subsidy" counterfactual, we calculate the gains from trade under different levels of a flat labor tax, used to finance a retraining subsidy. Unless governments can efficiently use tax revenue to overcome frictions to retraining, any positive tax rate makes workers worse off in the aggregate. The reason is simple: subsidies are poorly targeted because marginal retraining opportunities—retraining that would not have happened without the subsidies—are associated with relatively low benefits or high idiosyncratic costs. However, subsidies can benefit individual worker types even when decreasing aggregate welfare.

Related literature

Our paper links two strands of literature: labor market adjustment to trade shocks and worker retraining.

Trade adjustment An ever-growing set of papers, starting with Autor, Dorn, and Hanson (2013) and Autor et al. (2014), studies worker outcomes following a shock to sectoral import prices. Most relevant for our work, Dix-Carneiro (2014), Traiberman (2019), and Caliendo, Dvorkin, and Parro (2019) show that adjustment to shocks can be slow because of sector- or occupational-specific human capital and mobility barriers. Moreover, the welfare effects of trade vary widely by workers' (observed or unobserved) skill levels (Lee 2020; Traiberman 2019) and characteristics of their firm (Schott, Pierce, and Tello-Trillo 2020). Losses are spatially concentrated (Galle, Rodriguez-Clare, and Yi 2021) and become substantially more dispersed when correctly accounting for spatial linkages in general equilibrium (Adão, Arkolakis, and Esposito 2021). Trade opening and the China shock in particular have increased inequality and the skill premium, at least in high-income countries (Antràs, Gortari, and Itskhoki 2017; Lee 2020; Galle, Rodriguez-Clare, and Yi 2021), an effect accentuated by differences in occupational comparative advantage (Lee 2020; Traiberman 2019). On the other hand, there is relatively little work on retraining and trade. Hyman (2018) studies Trade Adjustment Assistance (TAA), a subsidized retraining program for import-competing workers in the US. Exploiting quasi-random variation in case examiner assignment, he shows that TAA benefits increase earnings.

In Germany, Dauth, Findeisen, and Suedekum (2014) and Dauth, Findeisen, and Suedekum (2021) find that, in aggregate, import competition from China and Eastern Europe in some sectors was offset by improved export opportunities in other sectors. The individual welfare effects of trade depend on how easily workers can transition to expanding sectors; a natural question to ask, then, is whether worker retraining programs can can speed workers' transitions toward better jobs and mediate the losses concentrated among certain groups.

Empirical retraining effects There is a large empirical literature in labor economics evaluating the causal effects of worker training programs. Kluve (2010) and Card, Kluve, and Weber (2010), in a pair of meta-analyses on active labor market policies, show that studies tend to report modest but positive employment effects, particularly in the medium to long-run. Papers tend to implement matching estimators (Fitzenberger and Speckesser 2007; Lechner and Wunsch 2009; Lechner, Miquel, and Wunsch 2011; Fitzenberger et al. 2013), or estimate models of dynamic selection into and treatment from retraining (Osikominu 2013; Biewen et al. 2014; Fitzenberger, Osikominu, and Paul 2021). The majority of papers report significant positive employment and earnings effects, with the latter result largely attributable to higher employment rates rather than higher earnings conditional on employment. Some papers, however, report null or even negative effects (Hujer, Thomsen, and Zeiss 2006; Lechner and Wunsch 2008; Doerr et al. 2017). Relative to this body of literature, our paper embeds retraining into a model of trade and frictional unemployment. We estimate the costs and benefits of retraining by expectation maximization and indirect inference. The parametrized model provides a clean framework for separating the effects of retraining on job finding from direct productivity effects, which is difficult in a reduced-form setting.

Methodologically, our paper builds on work in discrete choice models with unobserved heterogeneity (Arcidiacono and Miller 2011). We expand on the quasi-expectation maximization (EM) algorithm used by Traiberman (2019), allowing for time-variant unobserved types.

2 Background and Data

We briefly describe the data, introduce the institutional details, and lay out stylized facts about retraining. Section 3 connects retraining to labor market outcomes and shows that retraining is a margin of adjustment to import competition.

2.1 Data

The basis of the paper is the weakly anonymous Sample of Integrated Labor Market Biographies (SIAB), 1975-2017 (Antoni, Berge, and Ganzer 2019a).[2] The SIAB is a 2% sample of the German labor market drawn from the Integrated Employment Biographies (IEB). In turn, the IEB source data cover employment records, unemployment benefits, and, notably for our purposes, participation in active labor market programs (Antoni, Berge, and Ganzer 2019b).

We also use data on sectoral expenditure from the national accounts from the German federal statistical office (*Volkswirtschaftliche Gesamtrechnungen, Input-Output-Rechnung* 2014).

2.2 Retraining in Germany

2.2.1 Institutional details

The German Federal Employment Agency manages a variety of "active labor market policies" which overwhelmingly target unemployed workers (Dauth 2020; Fitzenberger, Osikominu, and Paul 2021).[3] Under the umbrella of retraining, we consider three categories of programs: *Activation and vocational integration*, *Career choice and vocational training*, and *Vocational retraining and further education*.[4] The SIAB includes retraining episodes starting in 2000.

Activation programs seek to match workers with jobs and provide necessary skills to (re)enter the labor market, ranging from job application training to foreign language classes (Bundesagentur für Arbeit 2021b; Roesler et al. 2021).

Career choice programs target primarily young workers, often still in school, providing

[2] The data were accessed on-site at the Research Data Centre (FDZ) of the Federal Employment Agency (BA) at the Institute for Employment Research (IAB) as well as remotely.

[3] Dauth 2020 discusses the lone subsidized training program for employed workers; however, only 0.1% of all employed workers have contact with the Federal Employment Agency in any given year.

[4] These correspond to the *erwstat* codes 10001 (activation and vocational integration), 10003 (career choice and vocational training), and 10007 (vocational retraining and further education) in the Participants-In-Measures History File (MTH/XMTH) portion of the SIAB.

career advising, government-sponsored internships, and tutoring to help complete vocational training (Bundesagentur für Arbeit 2021a; Roesler et al. 2021).

Further education, on the other hand, involves primarily the acquisition of skills. Such programs range from short courses teaching job search or computer skills to up to year-long training covering broad occupational skills such as marketing strategies to the acquisition of full 2-3 year vocational degrees (Osikominu 2013; Biewen et al. 2014)

Almost all training in the latter category is administered through a voucher system, whereby caseworkers use both need and chances of labor market success to award vouchers to workers that can be redeemed at any provider offering a specified type of retraining (Dauth 2020; Fitzenberger, Osikominu, and Paul 2021). Vouchers cover direct costs of training, transportation costs, lodging, and child care, as necessary. Workers also continue receiving unemployment insurance for the duration of the program (Bundesagentur für Arbeit 2021c). These courses typically combine some mix of classroom training with practical lessons or experience in firms (Osikominu 2013). Table 1 provides two examples of retraining curricula from the further education category.

Importantly, with the exception of the full vocational retrainings, these programs are supplementary to the standard vocational training system into which the majority of German students graduate. Retraining is targeted to help unemployed workers find and keep a job.

2.2.2 *Empirical facts*

Retraining is common in Germany: about twenty percent of workers have ever done retraining, and, as appendix figure A.1 further demonstrates, half of those have retrained more than once. Furthermore, these programs are prevalent across ages, education levels, and sectors (figures A.3-A.5).

Retraining courses are usually brief, with a median duration of 4 weeks (figure A.2). Furthermore, retraining in Germany is not primarily an on-the-job technology. Seventy-one percent of retraining participants are not employed in the month before starting retraining, and of those, three quarters do not find employment in the month after they finish (table A1). Even among those who are employed beforehand, almost half move to unemployment or a new industry after retraining (table A2).

The target group and scope of retraining, in our context, differ from those of standard notions of human capital investment such as attending college. Furthermore, the large literature on employer incentives to provide on the job training following Acemoglu and

Table 1: Retraining curriculum examples

Title	*Wind turbine training, compact*	*Solar energy training module 3: photovoltaic systems*
Length (weeks full time)	1	2
Prerequisites	vocational training in sales or technical area, computer skills	vocational training, 2 years' experience, and basic knowledge of solar technology.
Degree obtained	completion certificate	completion certificate
Cost with subsidy	0	0
Contents	• components of wind turbines • wind as energy source • yield and performance • examples of large wind parks • assembly • environmental effects • permitting process • profitability	• electrical fundamentals - principle of grid-connected system • stand-alone plants • planning and design • sizing • assembly • initial operation, maintenance • workplace safety

Source: Institut für Berufliche Bildung AG. Wind energy: https://www.ibb.com/weiterbildung/windkraftanlagen-kompakt#moreinformations and solar: https://www.ibb.com/weiterbildung/sonnenenergie-modul-3-photovoltaik-anlagen#moreinformations.

Pischke (1998) does not directly apply to the German context, where retraining is primarily done by the unemployed. Our work, then, provides empirical evidence and a theoretical lens through which to understand this distinct form of human capital investment.

3 Motivating facts

Having described what retraining means and what it looks like in the data, we now ask: why do workers retrain, and what happens once they do? First, we show that import-exposed workers are more likely to retrain. Second, we show that unemployed workers who retrain are more likely to (1) become re-employed (2) earn higher wages conditional on re-employment, and (3) switch sectors conditional on re-employment.

3.1 Import competition and retraining take-up

We present reduced-form evidence that retraining is a margin of adjustment to trade. Unemployed workers are significantly more likely to retrain in response to import competition. Imports are a widely used, measurable shock to labor demand.

3.1.1 Empirical strategy

For person-years during which a worker is *unemployed*, we estimate regressions of the form

$$\text{retrain}_{i,t+k} = \sum_{k=0}^{5} \beta_k b_k \times \Delta \log(\text{imports})_{s(i,t),t} + \sum_{k=0}^{5} b_k + W_{it}\Gamma + \gamma_{t+k} + \varepsilon_{itk} \tag{1}$$

where $\text{retrain}_{i,t}$ is a dummy for participating in subsidized retraining in year t, and $\Delta \log(\text{imports})_{s(i,t),t}$ is our key treatment variable: the change in log Eastern European and Chinese imports from $t-3$ to t in the sector in which the worker is observed in year 0, i.e. just before unemployment. The b_k are event study counter dummies for years of unemployment, where year 0 is the last year on the job before an unemployment spell. W_{it} is a vector of worker-level controls, and the γ_t are year fixed effects.[5]

Following the voluminous China shock literature (Autor, Dorn, and Hanson 2013; Dauth, Findeisen, and Suedekum 2014, 2021), we instrument for import growth from Eastern

[5]Demographic controls include age, gender, education, and foreign nationality. Controls characterizing the worker's year $k=0$ job include tenure, firm size, aggregated sector, skill level, log earnings, and year $t-3$ log sectoral imports and exports from Eastern Europe and China.

Europe and China to Germany using import growth in the time period and sector [6] from Eastern Europe and China to other high-income countries.[7] This instrument is valid under the assumption that the demand and supply shocks driving imports to other high-income countries are uncorrelated with German labor market conditions.

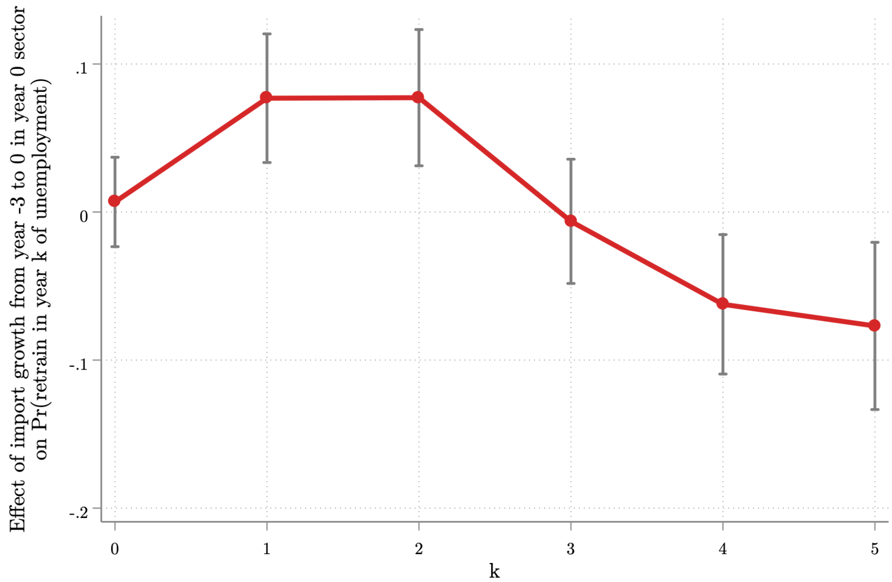
3.1.2 Results

Figure 1 plots the β_k: the effect of import growth in an unemployed worker's last sector on his/her probability of retraining in the kth of an unemployment spell. While import-exposed workers are no more likely to retrain in their last year on the job, these affected workers see a significant jump in their retraining probability in the first two years of unemployment. These coefficients imply that moving from the 25th to the 75th percentile of sectoral import exposure increases the probability that a worker losing a job in that sector will retrain in his/her first year of unemployment by around 3 percentage points. As a benchmark, about 7.5% of unemployed workers retrain in any given year.

[6] In other words, in the worker's year $k = 0$ sector $s(i,t)$ from year $t-3$ to t, i.e. the last three years prior to the beginning of unemployment.

[7] Specifically, the UK, Norway, Sweden, New Zealand, Australia, Japan, Canada, and Singapore.

Figure 1: Event study: retraining probability during unemployment



Source: SIAB. $N = 371,326$ person-years during unemployment. Year 0 is last year of employment before unemployment spell. Import growth in Germany is instrumented with import growth in other high-income countries. Demographic controls include age, gender, education, and foreign nationality, measured in year 0. Controls characterizing the worker's year 0 job include tenure, firm size, aggregated sector, skill level, log earnings, and year -3 log sectoral imports and exports from Eastern Europe and China. Net of calendar year k fixed effects.

3.1.3 Selection concerns

Instrumenting for imports does not, however, account for the fact that workers entering unemployment are a selected sample. To the extent that workers in non-import-exposed sectors lose their jobs due to low individual ability - rather than industrywide factors exogenous to individual characteristics - one might expect such workers to also have a lower return to retraining.

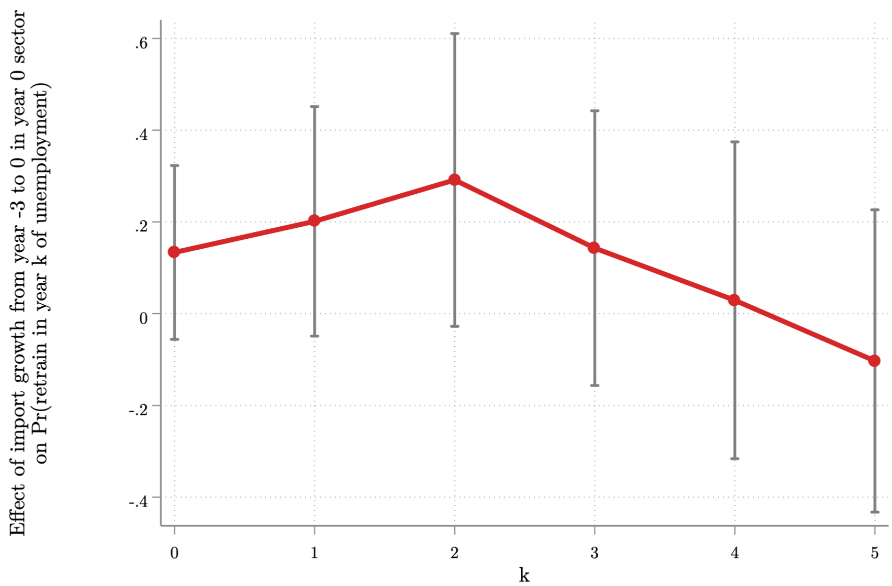
To address such concerns, we follow the labor literature in estimating the same event study regressions, restricting the sample to workers whose plant shuts down[8] in year t, the worker's last year on the job (Dauth and Eppelsheimer 2020). Presumably, the fact that such workers enter unemployment is unrelated to their own idiosyncratic ability or motivation.

Focusing on the plant closure workers (see Figure 2) similarly produces positive effects

[8]Specifically, whose establishment identifier appears in the full administrative dataset for the last time in year t.

of import exposure on retraining probability early in unemployment spells and zero or negative effects in the following years. While the estimates are noisier, due perhaps to the much smaller sample ($N = 11,938$ rather than $371,326$), the effect in year 2 is nevertheless significant at the 10% level.

Figure 2: EVENT STUDY: RETRAINING PROBABILITY DURING UNEMPLOYMENT FOLLOWING PLANT CLOSURE



Source: SIAB. $N = 11,938$ person-years during unemployment spells following establishment exit. Year 0 is last year of employment before unemployment spell. Import growth in Germany is instrumented with import growth in other high-income countries. Demographic controls include age, gender, education, and foreign nationality, measured in year 0. Controls characterizing the worker's year 0 job include tenure, firm size, aggregated sector, skill level, log earnings, and year -3 log sectoral imports and exports from Eastern Europe and China. Net of calendar year k fixed effects.

3.2 Retraining and labor market outcomes

Finally, we examine labor market outcomes post-retraining. The regression equation has a similar structure to (1). Consider a worker i who becomes unemployed in year t. For the k-th year of unemployment, each outcome y in the following year is modelled as

$$y_{i,t+k+1} = \beta \times \text{retrain}_{it} + \sum_{k=0}^{5} b_k + W_{it}\Gamma + \gamma_{t+k+1} + \varepsilon_{itk} \quad (2)$$

As before, W is a rich set of controls including demographics, tenure, previous earnings, lagged imports and exports of the previous sector, and the skill requirement of the previ-

Table 2: Retraining and labor market outcomes

Outcome	Employment	Earnings	Sector switch
	(1)	(2)	(3)
Subsidized retraining	0.053***	0.078***	0.213***
	(0.003)	(0.008)	(0.007)
Controls	✓	✓	✓
N	371,326	70,325	65,264
R^2	0.20	0.35	0.15

Source: SIAB. The sample is all years in which a worker was unemployed, for unemployment spells up to 5 years. Columns (2) and (3) are conditional on employment. Controls include year, age, sex, nativity, education, and duration of unemployment spell; tenure, log earnings, and skill level of previous job; size and coarse sector of previous firm; and three-year lagged imports and exports in detailed sector of previous firm. Standard errors in parentheses clustered at industry $\times$ year level (*** p<0.01).

ous job.

Table 2 shows estimates of (2) for three outcomes: a dummy for being employed next year, log earnings conditional on employment, and a dummy for switching to a new sector (relative to the sector before unemployment) conditional on employment.

Subsidized retraining is associated with a five percentage point increase in the probability of being employed in the subsequent year. Conditional on re-employment, earnings are eight percent higher and workers are twenty-one percentage points more likely to switch sectors. The baseline probability of switching sectors is about fifty-five percent for our unemployed sample, so the correlation between sector switching and retraining is economically significant.

3.3 Takeaway

A worker's decision to retrain is strongly predicted by import exposure. Subsequently, retraining is strongly associated with higher rates of re-employment and sector switching along with higher earnings. While we include a rich set of proxies for worker ability and exploit plausibly exogenous shocks like foreign imports and plant closures, retraining is endogenous to ability and preferences, both of which are unobserved. To provide a causal interpretation of retraining, we now turn to a structural model.

4 Theory

We now present a framework to rationalize the evidence on trade-induced retraining from Section 3 and to characterize how retraining changes the gains from trade. We start with the simplest possible model of a frictional labor market with heterogeneous workers. We then add retraining, which is a technology that allows workers to change their earnings potential and job-search abilities. The environment is stark. The only decision facing workers is whether and how to retrain.

4.1 Environment

The economy is populated by a unit mass of heterogeneous, infinitely-lived workers. Workers are characterized by a *type* $i \in 1, \ldots, I$ and can work in a *sector* $j \in 1, \ldots, J$. Superscripts index workers and function arguments index sectors.

The term "type" is deliberately generic. For concreteness, one can consider a dyad like $j \in \{\text{tradesman, generalist}\}$ or $j \in \{\text{blue collar, white collar}\}$. Our work marks a point of departure from the standard analysis of low- and high-skill workers. Types are unordered in the sense that type a may dominate b in one sector yet be worse in another. In our application, types will be inferred from the data rather than directly observed.

Each sector is an island with many identical firms facing exogenous world output price $p(j)$. The economy is small and all output is tradable. A worker of type i in sector j supplies $\epsilon^i(j)$ efficiency units of labor. Firms are perfectly competitive and operate at constant returns to scale, so type i workers in sector j earn $\epsilon^i(j)p(j)$. Employed workers face the consumption price index P.

Workers and sectors meet in frictional labor markets in continuous time. Unemployed workers have access to a backyard technology providing a flow b of real consumption. They match with sector j at rate $\lambda^i(j)$. Employed workers are separated at rate μ and transition to unemployment. There is no on-the-job search. Matching and separation rates are exogenous, and we assume parameter values such that all job offers are accepted. Workers have log utility over flows of real consumption and discount the future at rate r.

The novel feature of the model is retraining. As is typical of the data, retraining will be an investment available to unemployed workers. Unemployed workers receive random retraining shocks at rate $\varrho > 0$. Upon receiving a retraining shock, the worker can choose to transition to a new type subject to deterministic and idiosyncratic training costs. With exogenous probability ν, retraining fails after workers pay the training costs, in which case they retain their original type. Because retraining is usually brief—the median retraining spell lasts for about one month—we choose to model it as an instantaneous investment, after which workers return to unemployment.

Value functions

To ease the exposition, we focus on a steady state. Let U^i be the asset value of unemployment and $W^i(j)$ be the asset value of being employed in sector j. Let $R^i \geq U^i$ be the expected value of a retraining shock, to be described momentarily.

The value of unemployment is

$$rU^i = \log(b) + \sum_j \lambda^i(j)\left(W^i(j) - U^i\right) + \varrho\left(R^i - U^i\right). \tag{3}$$

Unemployed workers consume b and receive job offers at rate $\lambda^i(j)$, which they always accept by assumption. The aggregate job offer rate for type i is $\Lambda^i \equiv \sum_j \lambda^i(j)$. At rate ϱ, the unemployed receive retraining opportunities, to be described below.

The value of employment is the flow of real earnings discounted by the hazard of job loss,

$$rW^i(j) = \log\left(\frac{\epsilon^i(j)p(j)}{P}\right) + \mu\left(U^i - W^i(j)\right). \tag{4}$$

We assume that preferences over sectoral output are CES, so the CES price index is $P \equiv \left[\sum_j p(j)^{1-\phi}\right]^{\frac{1}{1-\phi}}$ and the expenditure share on sector j is $\eta(j) \equiv \left(\frac{p(j)}{P}\right)^{1-\phi}$. The assumption that the unemployment benefit b is in real terms ensures that values are homogeneous of degree zero in prices.[9]

Retraining

When an individual worker n of type i draws a retraining opportunity, he draws a vector of idiosyncratic retraining costs ζ_n^i, where $\zeta_n^{i\ell}$ represents the utility cost of moving from type i to type ℓ. In addition, he faces deterministic retraining costs $\kappa^{i\ell}$. We will impose

$$\kappa^{i\ell} = \begin{cases} \overline{\kappa} \geq 0 & \text{if } i \neq \ell \\ 0 & \text{otherwise} \end{cases}$$

[9] An alternative assumption would be to assume that b scales with real GDP, for example through a flat labor income tax.

The expected value of a retraining shock is

$$R^i \equiv \mathbb{E}\left[\max_{\ell}\left\{(1-\nu)\,U^{\ell}+\nu U^i-\kappa^{i\ell}+\zeta^{i\ell}\right\}\right].$$

Throughout the paper we will maintain the following assumption, standard in the discrete choice literature.

Assumption 1. *The idiosyncratic training costs, $\zeta_n^{i\ell}$, are independent random variables following a Type I Extreme Value distribution with zero mean and scale parameter $\sigma > 0$.*

Assumption 1 implies closed form expressions for the expected value of a retraining shock as well as the conditional probabilities of retraining to type ℓ from type i. The expected value of a retraining shock for type i satisfies

$$R^i = \sigma \log\left\{\sum_{\ell} \exp\left[(1-\nu)\,U^{\ell}+\nu U^i-\kappa^{i\ell}\right]^{1/\sigma}\right\}. \tag{5}$$

The probability of transitioning from type i to ℓ, conditional on drawing a retraining shock, is

$$\pi^{i\ell} = \frac{\exp\left[(1-\nu)\,U^{\ell}-\kappa^{i\ell}\right]^{1/\sigma}}{\sum_{\ell'}\exp\left[(1-\nu)\,U^{\ell'}-\kappa^{i\ell'}\right]^{1/\sigma}} \tag{6}$$

Labor market flows

The model satisfies standard accounting conditions for unemployment and labor supply. Let u^i denote the unemployment rate of type i workers. As a share of type i workers, flows from unemployment to employment are $u^i\Lambda^i$, which are balanced at every point in time by flows from employment to unemployment, $(1-u^i)\mu$. Hence the unemployment rate is

$$u^i = \frac{\mu}{\mu+\Lambda^i} \tag{7}$$

Given the way that we have chosen to model retraining, the standard accounting equation pinning down unemployment as a function of job-finding and job-losing rates still applies, type-by-type. This is because retraining does not affect job-finding or job-losing rates for a given type; instead, retraining can affect aggregate unemployment through the composition of types. Because separation rates do not vary by sector, the steady state employment share of type i in sector j is equal to the relative contact rate $\frac{\lambda^i(j)}{\Lambda^i}$.

Let τ^i be the steady-state share of workers of type i. The mass of unemployed of type

i, $\tau^i u^i$, are hit with retraining shocks at rate ϱ and either stay (with probability $\pi^{ii} + \nu \sum_{\ell \neq i} \pi^{i\ell}$) or move (with probability $(1-\nu)\sum_{\ell \neq i} \pi^{i\ell}$). At the same rate, a share $(1-\nu)\,\pi^{\ell i}$ of unemployed types ℓ move to type i. Thus, given the unemployment rates (7), the mass of types satisfies the I equations

$$\begin{aligned} \tau^i u^i \varrho \left[(1-\nu) \sum_{\ell \neq i} \pi^{i\ell} \right] &= \varrho \left[\sum_{\ell \neq i} \tau^\ell u^\ell \,(1-\nu)\, \pi^{\ell i} \right] \\ \Rightarrow \tau^i u^i &= \sum_{\ell} \tau^\ell u^\ell \pi^{\ell i} \end{aligned} \tag{8}$$

plus the adding-up constraint $\sum_i \tau^i = 1$.

We can now define an equilibrium.

Equilibrium. *An equilibrium consists of the value functions U^i and $W^i(j)$, retraining values R^i, transition probabilities $\pi^{i\ell}$, unemployment rates u^i, and type shares τ^i satisfying* (3), (4), (5), (6), (7), *and* (8).

Before moving on, we briefly discuss the rationale for presenting a stylized model which abstracts from life-cycle considerations, such as sector-specific experience, and does not feature endogenous job finding or job separation. These features are no doubt important, and have been explored in rich econometric models of labor supply (for example, in Dix-Carneiro 2014; Traiberman 2019). Our goal, instead, is to capture retraining in a way that is analytically tractable yet maintains contact with the empirical content from Section 3.

4.2 Characterization

Through retraining, workers gravitate toward types which have a comparative advantage in the labor market in terms of productivity $\epsilon^i(j)$ and search efficiency $\lambda^i(j)$, given the distribution of prices $p(j)$. Price changes will drive labor supply changes as workers choose to retrain. In this section we will formalize this intuition by describing labor supply, the response to shocks, and the gains from trade. To demonstrate the core ideas, we impose the following simplifications, which we will relax in the quantitative exercise in Section 5.

Assumption 2. *Job-finding rates do not differ by type ($\Lambda^i = \Lambda \;\; \forall i$), retraining is perfect ($\nu = 0$), and retraining is costless ($\overline{\kappa} = 0$).*

Under this assumption, the asset value of unemployment admits a very simple form. This is because the expected value of a retraining shock is equalized across types, which is immediate from inspection of (5). For convenience, define average log real earnings for type i by

$$y^i = \sum \lambda^i(j) \log \frac{\epsilon^i(j)p(j)}{P} \tag{9}$$

Substituting (4) into (3), we get the unemployment value as an affine function of earnings,

$$\begin{aligned} U^i &= (r+\mu)\vartheta\sigma \log b + \varrho\vartheta\sigma(r+\mu)R^i + \vartheta\sigma y^i \\ &= A + \vartheta\sigma y^i, \end{aligned} \tag{10}$$

where $\vartheta \equiv \left[\sigma\left((r+\varrho)(r+\mu)+r\Lambda\right)\right]^{-1}$ is a composite parameter and A collects terms not depending on i.

Next, zero retraining costs imply that the choice probabilities $\pi^{\ell i}$ are independent of the origin type ℓ. In other words, retraining decisions are determined only by destination gravity. The final implication of Assumption 2 is to equalize unemployment rates across types. Applying these two simplifications to the accounting constraint (8) and substituting the unemployment value (10) yields the steady-state type distributions:

$$\begin{aligned} \tau^i &= \pi^{\bullet i} \\ &= \frac{\exp\left[U^i\right]^{1/\sigma}}{\sum_\ell \exp\left[U^\ell\right]^{1/\sigma}} \\ &= \frac{\exp(\vartheta y^i)}{\sum_\ell \exp(\vartheta y^\ell)}, \end{aligned} \tag{11}$$

where we let $\pi^{\bullet i}$ denote the probability of transitioning to i from any origin type.

To build intuition for (11), let the discount rate r be small relative to the job finding rate Λ, job losing rate μ, and arrival rate of retraining shocks ϱ (as will hold in the calibrated model). Then, the long-run labor supply elasticity is $\vartheta \approx (\varrho\mu\sigma)^{-1}$. A high arrival rate of retraining shocks (ϱ) or a high variance of the retraining cost (σ) means that workers are less sensitive to wages because they will tend to change types frequently or for idiosyncratic reasons. A high job destruction rate (μ) makes workers less sensitive to expected earnings, since they spend less time employed.

Interpreting the labor supply curve

Expression 11 demonstrates that the steady state of our model is isomorphic to a static Roy model in which workers select into higher paying labor markets indexed by i. In the long run, individual retraining decisions aggregate to a stable economy-wide type distribution. Workers retrain for idiosyncratic reasons, but there are no net flows toward higher-paying types.

However, the short run looks very different. To clarify the dynamics of the model, consider a steady state hit with an arbitrary, exogenous change in world prices. The choice probabilities jump from $\pi^{\bullet i}$ to $\widehat{\pi}^{\bullet i}$ according to equation 11. This change is immediate because labor demand is perfectly elastic, so that wages jump to their new values right away. By contrast, labor supply adjusts slowly because retraining takes time. Along the transition path, the type shares at time t are

$$\tau_t^i = \widehat{\pi}^{\bullet i} + (\pi^{\bullet i} - \widehat{\pi}^{\bullet i}) \exp(-u\varrho t).$$

The speed of adjustment is governed by $u\varrho$, the share of workers receiving retraining shocks at each point in time. Convergence to the post-shock steady state is slow if u is small (unemployment, hence the pool of potential retrainees, is low) or if ϱ is small (conditional on unemployment, retraining opportunities shocks infrequently). Our framework is therefore equivalent not to a static Roy model, but rather to a dynamic one with slow adjustment to labor demand shocks.

4.3 Retraining and labor demand shocks

The regressions in Section 3.1 show that the retraining rate rises after an import shock to a worker's previous sector. To connect this empirical content to the model, we say that a worker of type i undertakes retraining if he receives a retraining shock and chooses a new type $\ell \neq i$. Retraining shocks that do not cause the worker to choose a new type are unobserved.

The object of interest is the hazard rate of retraining for all presently unemployed workers who previously worked in sector j. We will use the shorthand "j-unemployed" for this group. Let $\Xi(j) \equiv \sum_i \frac{\tau^i \lambda^i(j)}{\Lambda}$ denote the share of total employment in j. Since all workers lose and find jobs at the same rate, $\Xi(j)$ is also the share of j-unemployment out of total unemployment. The pool of j-unemployment is size $u \times \Xi(j)$, of which $u \times \frac{\tau^i \lambda^i(j)}{\Lambda}$ is type

i. Putting it all together, the retraining hazard for the j-unemployed is defined as

$$\begin{aligned}\mathcal{T}(j) &\equiv \varrho \frac{\sum_i \left(1-\pi^{\bullet i}\right) \tau^i u \frac{\lambda^i(j)}{\Lambda}}{\sum_\ell \tau^\ell u \frac{\lambda^\ell(j)}{\Lambda}} \\ &= \frac{\varrho}{\Lambda \Xi(j)} \sum_i \left(1-\pi^{\bullet i}\right) \tau^i \lambda^i(j). \end{aligned} \tag{12}$$

How does the retraining hazard respond to a small shock to the price of sector j output? Differentiating (12) with respect to the output price gives

$$\begin{aligned}\frac{\partial \mathcal{T}(j)}{\partial \log p(j)} &= -\frac{\varrho \vartheta}{\Lambda \Xi(j)} \sum_i \tau^i \lambda^i \left(\tau^i \lambda^i(j) - \tau^i \overline{\lambda}(j)\right) \\ &\leq 0 \end{aligned} \tag{13}$$

where $\overline{\lambda}(j) \equiv \sum_i \tau^i \lambda^i(j)$ is the aggregate rate of entry into j. The inequality on the second line of (13) follows from Jensen's inequality (see Appendix B for a short proof). In words, (13) says that a decline in the price of sector j output increases retraining among workers previously in j. The reason is that an individual's employment history is informative of their latent type. The difference in relative job-finding rates, $\lambda^i(j)/\lambda^\ell(j)$, is a source of comparative advantage in the model which determines workers' exposure to price changes.

The exercise also suggests that comparative advantage—consequently, specialization in employment—is necessary to rationalize trade-induced retraining. Indeed, if $\lambda^i(j)$ were equal across i, then the RHS of (13) would be zero. In this case, average log earnings are

$$\overline{y}^i = \underbrace{\sum_j \lambda(j) \log \epsilon^i(j)}_{\equiv \overline{\epsilon}^i} + \underbrace{\sum_j \lambda(j) \log \frac{p(j)}{P}}_{\equiv \overline{p}}$$

and the labor supply function reduces to

$$\overline{\tau}^i = \frac{\exp(\vartheta \overline{\epsilon}^i)}{\sum_\ell \exp(\vartheta \overline{\epsilon}^\ell)},$$

which is independent of prices. Without employment specialization, labor supply is invariant to trade.

Retraining and welfare

We close the discussion of the model by studying the welfare effects of labor market shocks. Of particular interest is how retraining changes the gains from trade.

First, we define aggregate steady-state welfare by

$$\mathcal{W} \equiv \sum_i \tau^i \left(u \left(rU^i \right) + (1-u) \sum_j \frac{\lambda^i(j)}{\Lambda} \left(rW^i(j) \right) \right) \tag{14}$$

Welfare is the weighted average of worker flow values across types and states, with weights given by the steady-state shares.

Now consider an arbitrary shock to world prices, $\Delta \log \boldsymbol{p}$. The instantaneous change in welfare, accounting both for the current effect of the price shock as well as the option value of retraining, is

$$\Delta \mathcal{W} \approx -\Delta \log P + \sum_j \Xi(j) \Delta \log p(j) + \frac{\mu \varrho \sigma}{\Lambda} D_{KL} \left(\boldsymbol{\pi} || \widehat{\boldsymbol{\pi}} \right)$$

where $\boldsymbol{\pi}$ and $\widehat{\boldsymbol{\pi}}$ are the pre- and post-shock choice probabilities, respectively. $D_{KL}(\boldsymbol{\pi}||\widehat{\boldsymbol{\pi}})$ is the Kullback–Leibler divergence from $\widehat{\boldsymbol{\pi}}$ to $\boldsymbol{\pi}$, defined by $D_{KL}(\boldsymbol{\pi}||\widehat{\boldsymbol{\pi}}) \equiv \sum_i \pi^{\bullet i} \log \frac{\pi^{\bullet i}}{\widehat{\pi}^{\bullet i}}$.

The first two terms are standard and reflect changes in the import price index and nominal earnings, respectively. Recall that $\Delta \log P \approx \sum_j \eta(j) \Delta \log p(j)$ for small shocks. A price increase in j is welfare-improving when the employment share $\Xi(j)$ is high relative to the consumption share $\eta(j)$.

The third term, $D_{KL}(\boldsymbol{\pi}||\widehat{\boldsymbol{\pi}})$, embeds the option value of retraining. The K-L divergence is a measure of spread between two distributions, and is always non-negative by construction. Unsurprisingly, the availability of retraining is never welfare-decreasing. The corollary is that retraining is most beneficial when labor market shocks create a large wedge between the initial and ideal type shares. Retraining will increase the gains from trade more, the larger the differences in worker comparative advantage across types.

5 Calibration

In this section we take the model to the data in order to address the counterfactual question of how retraining moderates the effects of labor market shocks. Ordinarily, the model could be estimated by maximum likelihood given data on wages, employment, and retraining episodes. However, we face an empirical challenge: worker types are unobserved. This necessitates a novel quantitative approach, which we call *Expectation Maximization with Dynamic Types* (EM-DT). As the name suggests, this is a modified Expectation Maximization (EM) algorithm in which worker types are permitted to change through observed retraining episodes. We now describe the implementation, give a broad overview of the algorithm, and discuss the estimates. A detailed description of the algorithm is relegated to Appendix C.

5.1 Data, notation, and model timing

Estimation requires taking a discrete-time approximation to the model. We assume the economy is in steady state. Workers, indexed by $n = 1, \ldots, N$, are observed in periods $t = 1, \ldots, T$. Each worker is characterized by an unobserved type $i(n,t)$ in each one-month period.

A worker enters period t with a state determined in $t-1$, either employed in a sector j or unemployed. It is convenient to divide each period into two stages At the start of the period, employed workers transition to unemployment with probability μ. Similarly, unemployed workers receive a job offer from sector j with probability $\lambda^{i(n)}(j)$. Job offers are disjoint. These transitions then determine the period t state.

During the period, employed workers produce. They receive an i.i.d. productivity shock ξ_{njt}, distributed normally with mean zero and standard deviation ς. The realized log wage is then $\log \omega_{njt} \equiv \log p_{jt} + \log \epsilon_j^{i(n,t)} + \xi_{njt}$. At the same time, unemployed workers receive a retraining opportunity with probability ϱ. Conditional on getting a retraining opportunity, they proceed to draw the retraining cost shocks $\{\zeta\}$, choose a type ℓ subject to retraining costs, and transition to type ℓ with probability $1-\nu$ or remain type i with probability ν.

To summarize, our estimator requires panel data for workers indexed n over T months indexed by t. Each period, we observe sector of employment dummy variables $\{d_{njt}\}_{n,j,t}$, employment dummies $\{e_{nt}\}_{n,t}$, retraining dummy variables $\{\rho_{nt}\}_{n,t}$ equal to 1 in periods where a worker is observed participating in retraining programs, and log residual earnings $\{\log \omega_{nt}\}_{n,t}$. We further define unemployment dummies $\{u_{nt}\}_{n,t}$ such that $u_{nt} \equiv 1 - e_{nt}$. Denote this dataset collectively as $D \equiv \left\{ \{d_{njt}\}_{n,j,t}, \{e_{nt}, u_{nt}, \rho_{nt}, \log \omega_{nt}\}_{n,t} \right\}$.[10]

[10]We fix the number of types to $I = 3$ and collapse the data to $J = 18$ sectors. Log wage residuals are taken after sweeping out fixed effects for time, sex, age, and immigrant status. We draw a balanced panel from 2013-2014 (length $T = 24$ months) with a 1% random sample of workers aged 25-55 with at most

Spells Explaining our algorithm requires defining the concept of a *spell*, i.e. the interval between retrainings during which a worker's type must remain constant. A worker always starts in spell 1. If he retrains, he moves to spell 2. A second retraining would be followed by spell 3, and so forth. Formally,

$$s_{nt} \equiv 1 + \sum_{t'<t} \rho_{nt} \tag{15}$$

and $s_{n0} = 1$, so that new spells always start in the period after retraining. Let $S(n)$ be the total number of observed spells for worker n.

Type probabilities The EM-DT uses *individual type probabilities*. Denote by $q_n^i(s) \equiv \Pr(i(n,s) = i)$ the probability that worker n is of type i in spell s.

5.2 Externally calibrated parameters

Some parameters are externally calibrated (see Table 3). The monthly discount rate r is set to 0.005. The elasticity of substitution between sectors, ϕ, is set to 4. The unemployment benefit b is equal to 60% of average monthly earnings. The job losing rate μ is set to 0.06. The standard deviation of the wage shock, ς, which governs the distribution of wages within sector-type, is set to 0.39 consistent with Traiberman (2019). The arrival rate of the retraining shock, ϱ, is not separately identified from the retraining cost $\overline{\kappa}$. Intuitively, data on realized retraining episodes cannot distinguish the frequent arrival of unappealing opportunities from the infrequent arrival of appealing ones. In the tradition of discrete choice models in discrete time (Artuç, Chaudhuri, and McLaren 2010; Caliendo, Dvorkin, and Parro 2019; Traiberman 2019, *inter alia*), we assume that workers on average receive one transition opportunity per year and set $\varrho = 0.08$. Finally, prices are not separately identified from the scale of productivity. We will estimate value added per worker denoted by $\varepsilon^i(j) \equiv \epsilon^i(j)p(j)$. The quantified model dispenses with Assumption 2, so that there are positive retraining costs and job finding rates are not necessarily equalized across types. For convenience, collect $\{\varepsilon^i(j)\}$ and $\{\lambda^i(j)\}$ into the vector Θ. The parameters to be estimated are Θ, ν, σ, and $\overline{\kappa}$.

a lower secondary school qualification. To reduce the computational burden, we drop the small number of workers who participate in multiple retraining episodes during the two years. The final sample size is $N = 202,370$ workers.

5.3 Overview of estimation algorithm

The estimation is a two-stage procedure. The first stage recovers the human capital parameters Θ and the retraining success parameter ν by EM-DT. The second stage recovers the retraining cost parameters $\overline{\kappa}$ and σ by indirect inference.

5.3.1 *First stage: human capital*

EM is a standard tool for estimating models with unobserved heterogeneity. In this class of models, heterogeneity is typically represented by a small number of types to keep the problem tractable. Suppose that types were observed, such that $q_n^{i*}(s)$ were a dummy equal to one if worker n is type i during spell s. Then, it would be straightforward to solve

$$\max_{\Theta} \log \mathscr{L}\left(\Theta; q^*, D\right) = \max_{\Theta} \sum_n \sum_i \sum_s q_n^{i*}(s) \log l_n^i\left(s, \Theta; D\right), \tag{16}$$

where $l_n^i(s,\Theta;D) \equiv \prod_{t:s_{nt}=s} l_{nt}^i\left(\Theta;D\right)$ is the likelihood of spell s when n is type i. The likelihood can be written explicitly as a function of data, parameters, and the probability of choosing to retrain. While the retraining probability is the endogenous solution to a dynamic problem, we can instead substitute its empirical counterpart conditional on knowing q^* (e.g., the observed retraining frequency for each group i).

The key idea of EM is to estimate the unobserved type index q^* by Bayes' rule. Initialize the algorithm with a guess $q^{(0)}$ and $\Theta^{(0)}$. Then, at the m-th iteration,

1. Compute the frequency of retraining among type i, $\hat{\delta}^{i,(m)}$, equal to the q-weighted average of observed frequencies.
2. Evaluate the likelihood $l_n^{i,(m)}(s,\Theta^{(m)};D)$.
3. Update q according to

$$q_n^{i,(m+1)}(s) = \frac{\overline{q}^{i,(m)} l_n^{i,(m)}(s,\Theta^{(m)};D)}{\sum_\ell \overline{q}^{\ell,(m)} l_n^{\ell,(m)}(s,\Theta^{(m);D})},$$

   where $\overline{q}^{i,(m)}$ is the prior for q.

   - For the workers who do not retrain, the prior is just the population type share $\tau^{i,(m)}$, equal to the sample average of $q_n^{i,(m)}(s)$.
   - For the retrainers, the prior is the previous type index $q_n^{i,(m)}(s-1)$ weighted by the empirical transition probabilities $\hat{\omega}^{i\ell}$ (which are equal to the q-weighted

average frequency of switching from i to ℓ; see Appendix for details)

4. Solve for $\Theta^{(m+1)}$ by maximizing (16), replacing the unknown $q_n^{i*}(s)$ with the guess $q_n^{i,(m+1)}(s)$.

5. Return to step 1 and repeat until convergence.

In simple terms, the posterior q is computed from the prior q and a parameter guess. Then, the parameters are updated using the posterior q. Once the algorithm converges, the type probabilities q can be used to recover the retraining failure parameter ν as well as the empirical retraining choice matrix $\hat{\pi}^{i\ell}$. See Appendix C for details.

5.3.2 *Second stage: retraining*

The remaining parameters σ and $\overline{\kappa}$ are estimated in a second stage by indirect inference. The first stage pins down the full matrix of empirical choice probabilities $\hat{\pi}^{i\ell}$. In words, this is the probability, conditional on receiving a retraining shock, that a type i worker attempts to retrain to type ℓ. The first stage estimates also yield average log income for each type, $\hat{y}^{\ell}$.

We regress the empirical choice probabilities $\log \hat{\pi}^{i\ell}$ on estimated destination income $\hat{y}^{\ell}$, a dummy equal to 1 if $i \neq \ell$ to capture moving costs, and a set of origin fixed effects. Call the regression coefficients on income and the moving cost $\hat{\beta}_y$ and $\hat{\beta}_m$, respectively.

For any guess of σ and $\overline{\kappa}$, we can solve the model and run the same regression to obtain $\hat{\beta}_y^{model}$ and $\hat{\beta}_m^{model}$.[11] The second stage of the estimation selects the unique values of σ and $\overline{\kappa}$ such that the model-implied regression coefficients exactly match their counterparts from the data.

5.4 Discussion of estimates

Table 3 summarizes our calibrated parameters. We have already discussed the externally calibrated parameters in section 5.2.

[11] Average log income in the model is simply read off the first stage estimates, but the model-implied choice probabilities need not match exactly.

Table 3: Parameters

Parameter	Description	Value
Externally calibrated		
I	number worker types	3
J	number sectors	18
r	rate of time preference	0.005
b	unemployment benefit	0.732
μ	separation rate	0.06
ϱ	rate of retraining shocks	0.08
ς	wage shock std. deviation	0.3873
ϕ	sectoral elasticity of sub.	4
1st stage		
ν	Pr. of retraining failure	0.36
$\lambda^i(j)$	type i job offer arrival rate from sector j	see Table A3
$\varepsilon^i(j)$	type i value added per worker in sector j	see Table A4
2nd stage		
σ	variance of idiosyncratic type pref. shocks	1.69
$\bar{\kappa}$	retraining cost	4.499

The first stage yields a retraining failure probability $\hat{\nu}$ over one third. Thus, retraining is a somewhat risky investment for workers. The fact that retraining, in the data, often does not seem to substantially change workers' job finding and earnings might suggest significant room for improvement in the quality of programs.

The second stage procedure gives a idiosyncratic type preference shocks variance scaling factor of $\sigma = 1.69$. This variance scaling factor is inversely related to the elasticity of retraining probabilities with respect to deterministic values and costs, with a higher σ indicating lower sensitivity to the costs and benefits of retraining. The value is slightly lower than benchmark values from the literature (Caliendo, Dvorkin, and Parro (2019) estimate a value of 5.34 for sectoral-location choice while Artuç, Chaudhuri, and McLaren (2010) report values between 2 and 4 for sector choice). The parameter is not directly comparable because it governs the retraining decision rather than sector choice directly.

We estimate, also in the second stage, a retraining cost of $\bar{\kappa} = 4.50$ to transition into other types. To interpret the magnitude of this number, consider the thought experiment of taking $\bar{\kappa}$ to zero while simultaneously reducing the unemployment benefit b such that (*ex-ante*) aggregate welfare is unchanged. The unemployment benefit declines by 8.8%: substantial, but not prohibitive.

The most interesting parameters characterize the three unobserved worker types, namely the job offer arrival rates $\lambda^i(j)$ and the value-added per worker $\varepsilon^i(j)$ by sector, shown in Figures 3 and 4 or Appendix Tables A3 and A4, respectively.

Type 1 workers have high (unemployed) job offer arrival rates from all sectors, as figure 3 demonstrates; once employed, their value added per worker is often lies between that of the other two types (figure 4). Thus, we could label type 1 workers *generalists*.

Type 2 workers, on the other hand, have the lowest job offer arrival rates in most sectors. However, they have an absolute advantage in production in the majority of sectors, especially fuel and chemicals, auto, and other transport manufacturing, industries in which Germany is generally thought to have comparative advantage. Thus, we label type 2 *skilled tradespeople*.

Finally, type 3 workers receive job offers at relatively average rates; their value added is generally lower than the skilled tradespeople, but they have a clear absolute advantage over the generalists in auto and transport manufacturing. Consequently, label them *autoworkers*.

Figure 3: JOB OFFER ARRIVAL RATE ESTIMATES, $\lambda^i(j)$

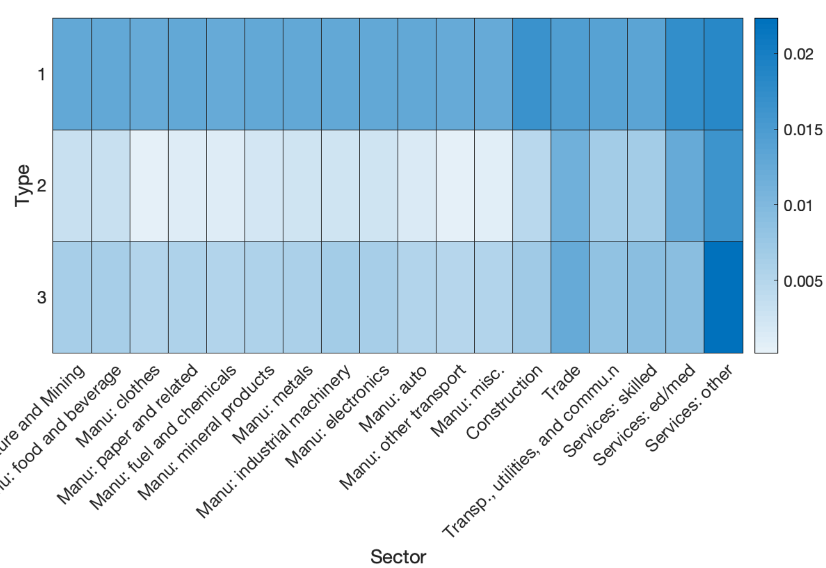
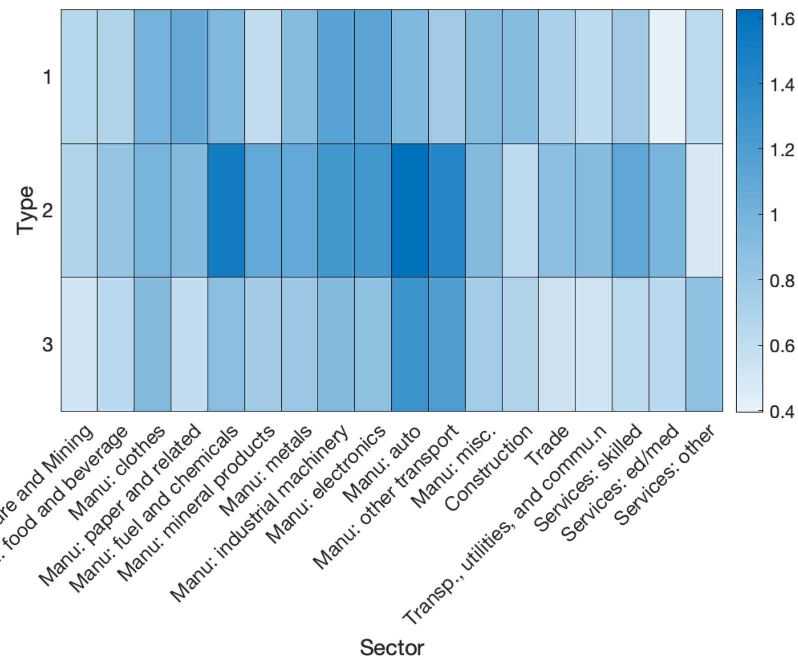


Figure 4: VALUE ADDED PER WORKER ESTIMATES, $\varepsilon^i(j)$



Summary statistics for the three types are presented in Panel A of Table 4. Skilled tradespeople have the highest incomes and make up over half of our noncollege sample. Autoworkers account for another third and have slightly lower incomes. Generalists, the smallest group, tend to find jobs at high rates but have low incomes on average. Panel B shows the retraining choice probabilities from the data. The matrix is diagonal dominant, as most retraining opportunities are not taken up. But conditional on receiving an opportunity, generalists are most likely retrain (across the first row), unsurprising given their low earnings potential. The skilled trades, with the highest earnings potential, tend to attract retrainers (down the second column).

Table 4: Type characterization

	Worker type (i)		
Panel A: Characteristics	Generalists	Skilled Trades	Autoworkers
Type shares τ^i	0.1516	0.5269	0.3215
Job finding rate Λ^i	0.2461	0.0786	0.1359
Expected income y^i	-0.0607	-0.0150	-0.0417
Panel B: Retraining choice π	Generalists	Skilled Trades	Autoworkers
Generalists	0.7616	0.1464	0.0921
Skilled Trades	0.0230	0.9148	0.0623
Autoworkers	0.0240	0.0955	0.8805

Note: τ and π estimated directly from the distribution of types in the data. $\pi^{i\ell}$ is depicted in the ith row and ℓth column.

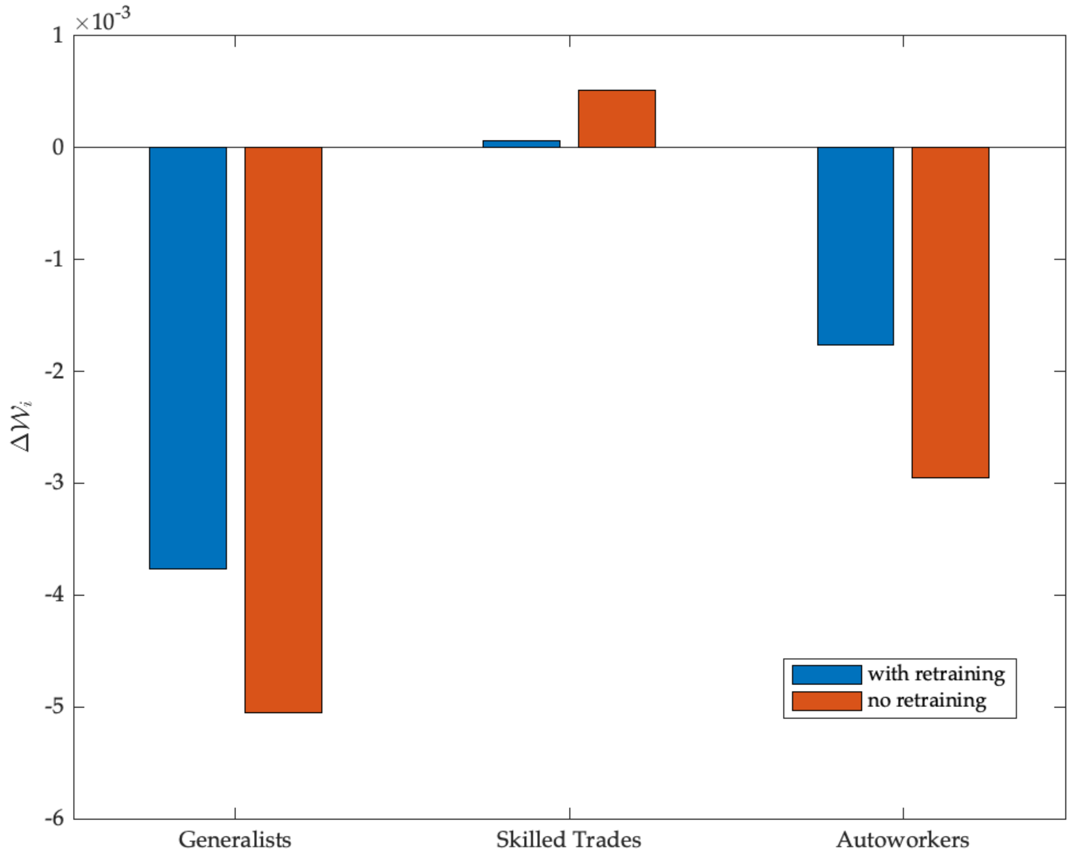
6 Counterfactuals

The simple model validated the intuition that retraining tends to increase the gains (or decreases the losses) from trade. To quantify the importance of retraining, we use the estimated model to study the effects of a counterfactual shock to import prices.

Starting from the steady state, we decrease the output prices in the clothing sector and the electronics sector each by ten percent. Then we compute the change in welfare at impact according to (14), weighting by type shares at the initial steady state. As suggested by the decomposition (15), the welfare effect of the shock is negative because it is biased toward employment rather than consumption. Clothing and electronics together comprised 5.4% of employment but just 4.2% of consumption expenditure in Germany in 2014. Welfare declines by 0.11% in the baseline case and 0.12% when workers are not permitted to retrain (i.e. in the model with $\varrho = 0$), which is a difference of about seven percent.

The welfare effects differ substantially by type. As the only type not specialized in the auto industry and related sectors, generalists lose the most from the trade shock to clothing and electronics; allowing them to retrain reduces the welfare loss by 25 percent. Skilled tradesmen actually see a much larger welfare increase in a world without retraining, as their pre-trade welfare is already substantially higher when allowed to retrain. Interestingly, despite the fact that their comparative advantage sectors are unaffected, au-

Figure 5: Trade shock: change in welfare by worker type



toworkers see the largest gain from retraining, at least in relative terms: their "losses from trade" fall by almost 40 percent when they have the option to retrain.

These distributional effects are reflected in the path of the skill distribution is displayed in Figure 6. Type 1 is the most adversely affected, with 10% of type 1 workers employed in the import-competing sectors. Workers tend to gravitate toward type 2, which has an employment share of under 3.5% in the two sectors. The adjustment is very slow. While half of net flows are completed within two years, the full transition takes over a decade. This is not too surprising because retraining is relatively infrequent. In our data, between 0.1% and 0.2% of workers retrain in any given month.

6.1 Retraining subsidies

The model rationalizes infrequent retraining with a high retraining cost. In this section we consider whether an additional subsidy to retraining can effectively surmount this obstacle.

In particular, suppose there is a government which has access to a technology which reduces frictions to retraining. This could be interpreted as increasing payments to workers, raising awareness of program options, or simply improving program design. The government can spend g per worker in order to reduce the retraining cost by ψg, where ψ parametrizes the effectiveness of government intervention. A high ψ means that public expenditure strongly reduces barriers to retraining. The payment g is funded with a flat labor income tax rate m, chosen so that the government budget balances.

Figure 7 plots the change in welfare under the counterfactual price shock against the generosity of the subsidy, for several different efficiency levels ψ. We present the results in terms of the tax rate. To benchmark these numbers, note that a 2.5% tax rate given $\psi = 1$ implies a retraining subsidy equal to about half of the estimated $\bar{\kappa}$. The green line is the baseline discussed in the prior subsection, with no subsidy policy. Of course, for any given tax rate, a more efficient subsidy is better. What is more surprising is that subsidies are almost always socially wasteful in the aggregate. Only an extremely efficient subsidy—the purple line—improves welfare, and even then at relatively low levels of funding. This is not because the subsidies are moot. Retraining subsidies do increase retraining take-up. For example, with $\psi = 1$ and a tax rate of 2.5%, the average retraining hazard $\mathcal{T}$ is slightly under 2%, compared with about 0.7% without taxes. Rather, the subsidy scheme is undone by selection, because the marginal retraining opportunity is less profitable than the average retraining opportunity. Under a 2.5% tax rate, the average increase in expected income following retraining (inclusive of unemployment benefits) falls by 30% relative to a no-tax régime. In other words, subsidies make retraining opportunities enticing even if the expected income gain is low.

However, the distributional effects of subsidies are more subtle; a larger range of subsidy efficiencies and tax rates decrease autoworkers' losses from trade (or indeed even turn them positive). Only the most efficient retraining subsidies could increase generalists' and skilled tradesmen's gains from trade. Thus, interestingly, retraining subsidies do not do much to help those workers–the generalists–most hurt by the trade shock. Because they are infrequently unemployed, generalists pay the tax that funds other workers' retraining yet rarely receive retraining opportunities themselves.

Figure 6: Type shares after shock to import prices

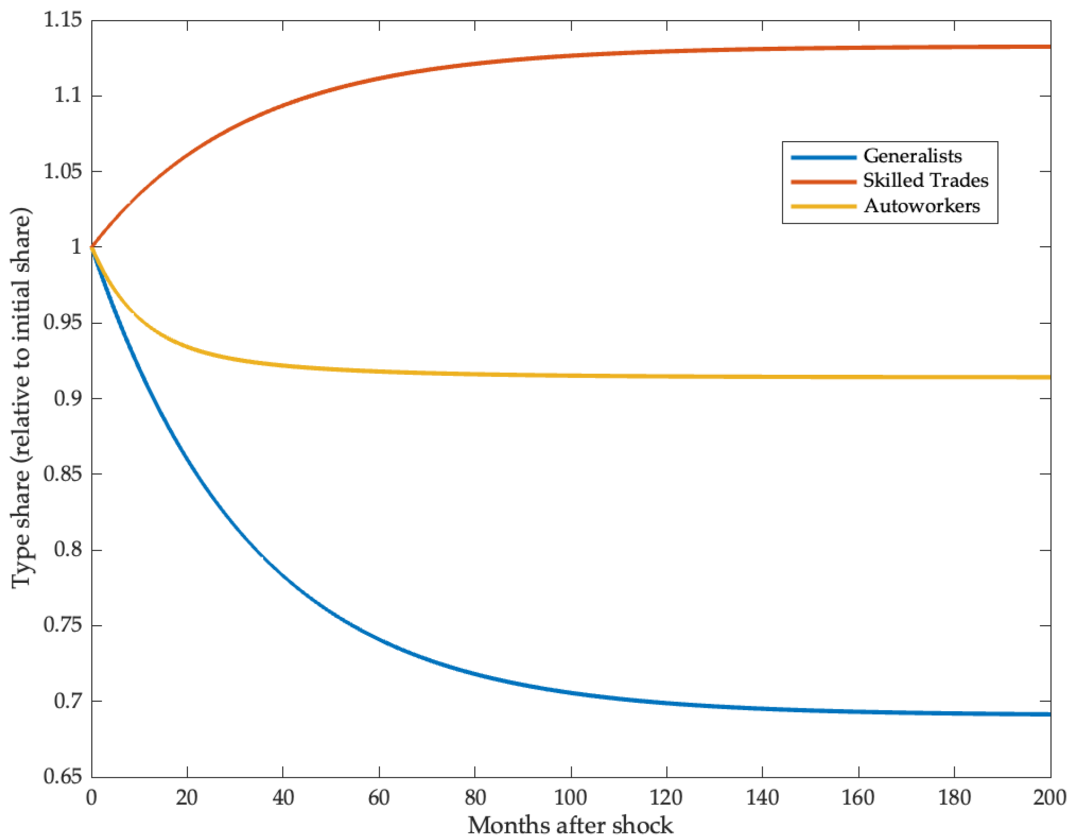


Figure 7: Retraining subsidy: change in welfare from shock (high ψ is more effective subsidy)

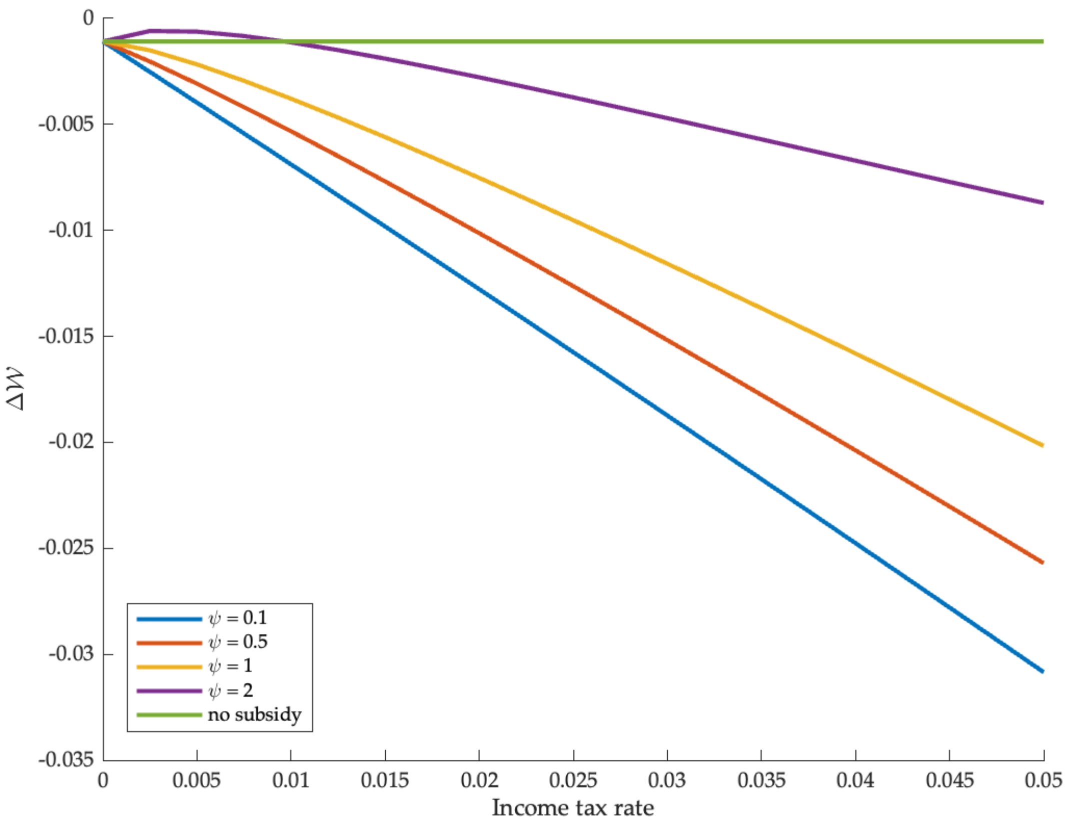
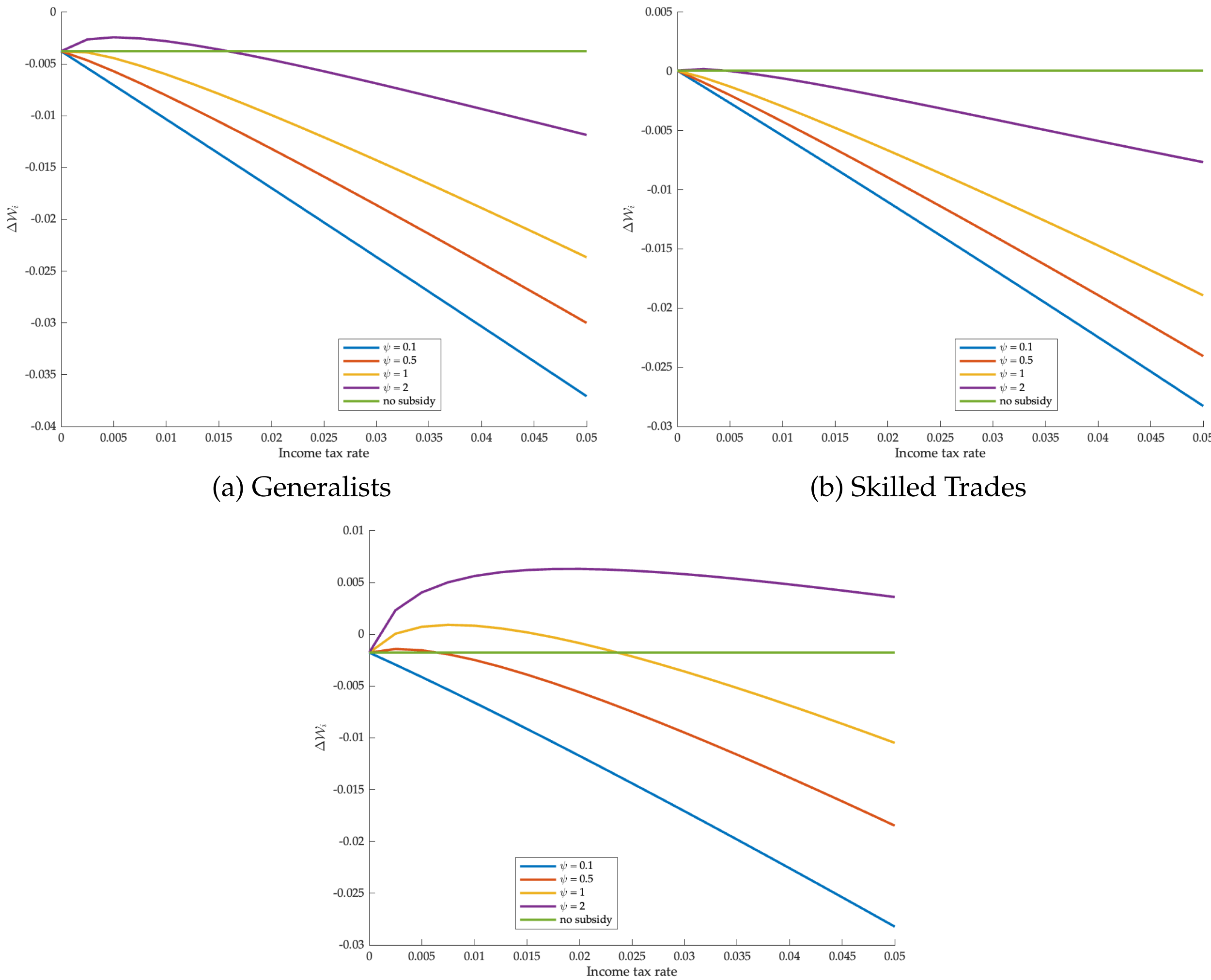


Figure 8: RETRAINING SUBSIDY: CHANGE IN WELFARE FROM SHOCK BY TYPE



(a) Generalists

(b) Skilled Trades

(c) Autoworkers

7 Conclusion

We have provided evidence and some theory that workers in Germany retrain in response to trade shocks. The more specialized a worker type is in a given sector, the greater that type's comparative advantage in the sector and thus the larger the gains from switching types after a sectoral import shock.

By allowing workers to redirect their skills to better match the most profitable sectors, retraining increases the gains from trade. The gains from retraining are large when the original and the ideal labor supply curves are dissimilar. In our counterfactual, the magnitude of the difference is about 7%, a statistic which masks considerable heterogeneity. The groups who gain the least from trade are not the ones who benefit the most from the opportunity to retrain. We find that government programs that efficiently remove frictions to retraining can have sizable welfare gains. We leave to future work the task of specifying a richer model of retraining costs and tax distortions that can facilitate the design of such policies.

References

Acemoglu, Daron, and Jörn-Steffen Pischke. 1998. "Why Do Firms Train? Theory and Evidence." *The Quarterly Journal of Economics* 113 (1): 79–119.

Adão, Rodrigo, Costas Arkolakis, and Federico Esposito. 2021. "General Equilibrium Effects in Space: Theory and Measurement."

Alden, Edward. 2016. "Column: How to help workers laid low by trade — and why we haven't." *PBS* (November 16, 2016). https://www.pbs.org/newshour/economy/column-help-workers-laid-low-trade-havent.

Antoni, Manfred, Philipp vom Berge, and Andreas Ganzer. 2019a. *Factually anonymous Version of the Sample of Integrated Labour Market Biographies (SIAB-Regionalfile) – Version 7517 v1.* Research Data Centre (FDZ) of the Federal Employment Agency (BA) at the Institute for Employment Research (IAB). https://doi.org/10.5164/IAB.SIAB-R7517.de.en.v1.

———. 2019b. *Sample of Integrated Labour Market Biographies Regional File (SIAB-R) 1975 - 2017, FDZ-Datenreport.* Technical report. Nürnberg. https://doi.org/10.5164/IAB.FDZD.1904.en.v1.

Antràs, Pol, Alonso de Gortari, and Oleg Itskhoki. 2017. "Globalization, inequality and welfare." *Journal of International Economics* 108:387–412.

Arcidiacono, Peter, and Robert A Miller. 2011. "Conditional choice probability estimation of dynamic discrete choice models with unobserved heterogeneity." *Econometrica* 79 (6): 1823–1867.

Artuç, Erhan, Shubham Chaudhuri, and John McLaren. 2010. "Trade shocks and labor adjustment: A structural empirical approach." *American Economic Review* 100 (3): 1008–45.

Autor, David, David Dorn, and Gordon H Hanson. 2013. "The China syndrome: Local labor market effects of import competition in the United States." *American Economic Review* 103 (6): 2121–68.

Autor, David H, David Dorn, Gordon H Hanson, and Jae Song. 2014. "Trade adjustment: Worker-level evidence." *The Quarterly Journal of Economics* 129 (4): 1799–1860.

Biewen, Martin, Bernd Fitzenberger, Aderonke Osikominu, and Marie Paul. 2014. "The effectiveness of public-sponsored training revisited: The importance of data and methodological choices." *Journal of Labor Economics* 32 (4): 837–897.

Bundesagentur für Arbeit. 2021a. "Ausbildung vorbereiten und unterstützen." https://www.arbeitsagentur.de/bildung/ausbildung/ausbildung/ausbildung-vorbereiten-unterstuetzen.

———. 2021b. "Diese Maßnahmen fördert die Bundesagentur für Arbeit." https://www.arbeitsagentur.de/bildungstraeger/diese-massnahmen-foerdert-die-ba.

———. 2021c. *Förderung der beruflichen Weiterbildung für Arbeitnehmerinnen und Arbeitnehmer.* Technical report. https://www.arbeitsagentur.de/datei/merkblatt-6-weiterbildung_ba015381.pdf.

———. 2021d. *Haushaltsplan 2021.* Technical report. https://www.arbeitsagentur.de/datei/nhp-2021_ba146853.pdf.

Caliendo, Lorenzo, Maximiliano Dvorkin, and Fernando Parro. 2019. "Trade and labor market dynamics: General equilibrium analysis of the china trade shock." *Econometrica* 87 (3): 741–835.

Card, David, Jochen Kluve, and Andrea Weber. 2010. "Active labour market policy evaluations: A meta-analysis." *The economic journal* 120 (548): F452–F477.

Dauth, Christine. 2020. "Regional discontinuities and the effectiveness of further training subsidies for low-skilled employees." *ILR Review* 73 (5): 1147–1184.

Dauth, Wolfgang, and Johann Eppelsheimer. 2020. "Preparing the sample of integrated labour market biographies (SIAB) for scientific analysis: a guide." *Journal for Labour Market Research* 54 (1): 1–14.

Dauth, Wolfgang, Sebastian Findeisen, and Jens Suedekum. 2014. "The rise of the East and the Far East: German labor markets and trade integration." *Journal of the European Economic Association* 12 (6): 1643–1675.

———. 2021. "Adjusting to globalization in Germany." *Journal of Labor Economics* 39 (1): 263–302.

Dix-Carneiro, Rafael. 2014. "Trade liberalization and labor market dynamics." *Econometrica* 82 (3): 825–885.

Doerr, Annabelle, Bernd Fitzenberger, Thomas Kruppe, Marie Paul, and Anthony Strittmatter. 2017. "Employment and earnings effects of awarding training vouchers in Germany." *ILR Review* 70 (3): 767–812.

Fitzenberger, Bernd, Olga Orlanski, Aderonke Osikominu, and Marie Paul. 2013. "Déjà Vu? Short-term training in Germany 1980–1992 and 2000–2003." *Empirical Economics* 44 (1): 289–328.

Fitzenberger, Bernd, Aderonke Osikominu, and Marie Paul. 2021. "The Effects of Training Incidence and Duration on Labor Market Transitions."

Fitzenberger, Bernd, and Stefan Speckesser. 2007. "Employment effects of the provision of specific professional skills and techniques in Germany." *Empirical Economics* 32 (2): 529–573.

Friedman, Jerome, Trevor Hastie, and Robert Tibshirani. 2008. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction.* 2nd ed. New York: Springer.

Galle, Simon, Andres Rodriguez-Clare, and Moises Yi. 2021. "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade."

Hujer, Reinhard, Stephan L Thomsen, and Christopher Zeiss. 2006. "The effects of vocational training programmes on the duration of unemployment in Eastern Germany." *Allgemeines Statistisches Archiv* 90 (2): 299–321.

Hyman, Benjamin. 2018. "Can Displaced Labor Be Retrained? Evidence from Quasi-Random Assignment to Trade Adjustment Assistance."

Kluve, Jochen. 2010. "The effectiveness of European active labor market programs." *Labour Economics* 17 (6): 904–918.

Lechner, Michael, Ruth Miquel, and Conny Wunsch. 2011. "Long-run effects of public sector sponsored training in West Germany." *Journal of the European Economic Association* 9 (4): 742–784.

Lechner, Michael, and Conny Wunsch. 2008. "What Did All the Money Do? On the General Ineffectiveness of Recent West German Labour Market Programmes." *KYKLOS International Review for Social Sciences* 61 (1): 134–174.

———. 2009. "Are training programs more effective when unemployment is high?" *Journal of labor economics* 27 (4): 653–692.

Lee, Eunhee. 2020. "Trade, inequality, and the endogenous sorting of heterogeneous workers." *Journal of International Economics* 125:103310.

Morath, Eric. 2018. "Retraining Programs Fall Short for Some Workers." *The Wall Street Journal* (December 30, 2018). https://www.wsj.com/articles/retraining-programs-fall-short-for-some-workers-11546088401.

Osikominu, Aderonke. 2013. "Quick job entry or long-term human capital development? The dynamic effects of alternative training schemes." *Review of Economic Studies* 80 (1): 313–342.

Parlapiano, Alicia, and Quoctrung Bui. 2018. "Everything in the House Democrats' Budget Bill." *The New York Times* (November 18, 2018). https://www.nytimes.com/interactive/2021/11/18/upshot/whats-in-democrats-reconciliation-bill.html?campaign_id=9&emc=edit_nn_20211119&instance_id=45793&nl=the-morning&regi_id=82269758&segment_id=74826&te=1&user_id=ed7501af33e3bb4287c6ef8a5bea3080.

Roesler, Konrad, Christiane Papenroth, Antje Hess, Sylke Gollin, Joachim Fritz, Angela Kahler, Sebastian Lorenz, and Sylvie Böhme. 2021. *Statistik zu Maßnahmen und Teilnehmenden an Maßnahmen der Arbeitsförderung.* Technical report. Bundesagentur für Arbeit. https://statistik.arbeitsagentur.de/DE/Statischer-Content/Grundlagen/Methodik-Qualitaet/Qualitaetsberichte/Generische-Publikationen/Qualitaetsbericht-Statistik-Massnahmen-Teilnehmer-Arbeitsfoerderung.pdf?__blob=publicationFile.

Schott, Peter, Justin Pierce, and Cristina Tello-Trillo. 2020. "Trade Liberalization and Labor-Market Outcomes: Evidence from US Matched Employer-Employee Data."

Traiberman, Sharon. 2019. "Occupations and import competition: Evidence from Denmark." *American Economic Review* 109 (12): 4260–4301.

Volkswirtschaftliche Gesamtrechnungen, Input-Output-Rechnung. 2014. Statistisches Bundesamt.

A Empirical appendix

A.1 Facts About Retraining

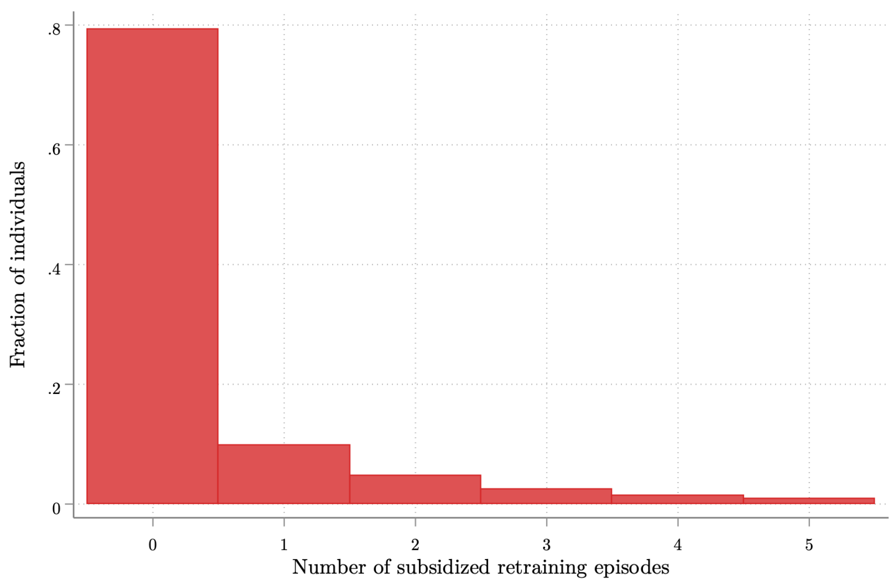
To build intuition for what retraining looks like in the data, we document a set of facts. We focus on the extensive margin of retraining, defining retraining as a dummy variable equal to one if the worker started a retraining program at any point in the year.

First, we turn to the frequency and duration of retraining.

Fact 1. *Retraining is repeatable and usually brief.*

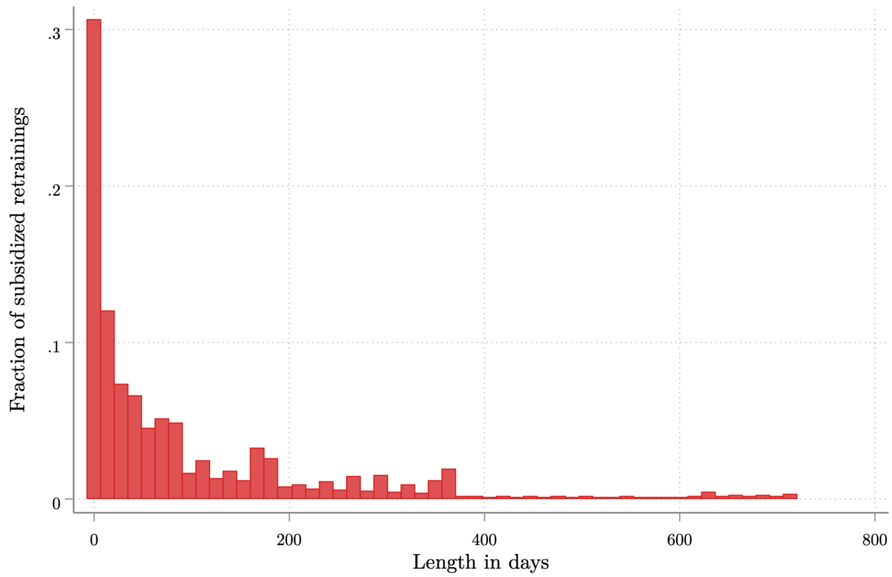
About twenty percent of workers have ever done retraining. Of that, about half—ten percent of the workforce—has retrained more than once. This is consistent with a relatively low time cost of retraining. Retraining episodes tend to be brief, although with substantial heterogeneity. Figure A.2 plots the distribution of episode durations. The median retraining episode lasts four weeks, with the 25th percentile being two weeks and the 75th percentile sixteen weeks.

Figure A.1: LIFETIME RETRAINING PROBABILITIES



Source: SIAB.

Figure A.2: DURATION OF RETRAINING EPISODES



Source: SIAB.

Next, we turn to the demographics of retraining.

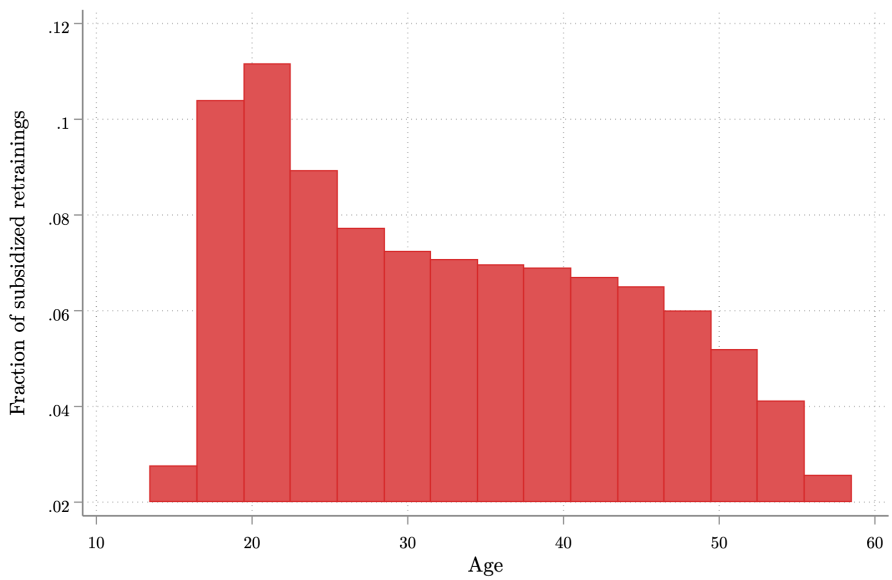
Fact 2. *Workers retrain at all ages, education levels, and sectors.*

Figure A.3 shows age at retraining. The largest share of retraining is done by the young, which no doubt partly reflects higher unemployment rates among that cohort. However, while retraining is slightly declining in age, it remains relatively common through age 50. Unlike vocational training or university, retraining is a form of continuing education.

Next, retraining is not restricted to low-skill workers. Figure A.4 shows the lifetime probability of retraining at least once by education category. Workers with a vocational degree—constituting the majority of the German labor force—have the highest take-up, with about one quarter having ever participated in retraining. Take-up is lowest among university graduates, but still appreciable. Workers without any post-secondary education, which label "No vocational degree," are in the middle.

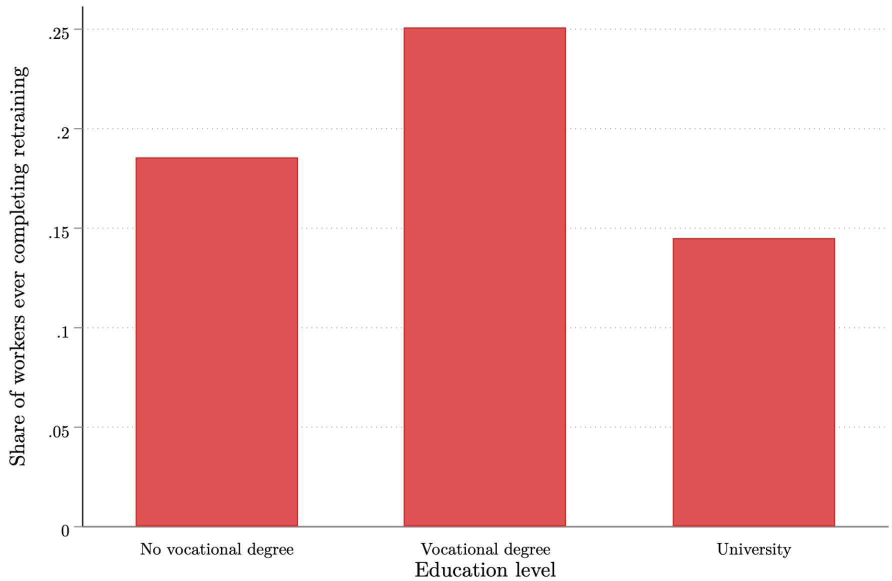
A similar pattern holds by sector, among workers employed in the year of retraining. Concretely, we assign each worker the sector of their employer on June 30th of each year. If a worker has more than one job, the highest paying job is selected. Figure A.5 shows the distribution of employment among workers who retrain (right) and do not retrain (left), split by aggregate sector. The distributions are similar. Workers in construction retrain at slightly higher rates, and workers in high-tech manufacturing retrain at slightly lower rates. The picture is virtually unchanged if we classify workers by their sector in the year following retraining rather than the year of retraining.

Figure A.3: Age at Retraining



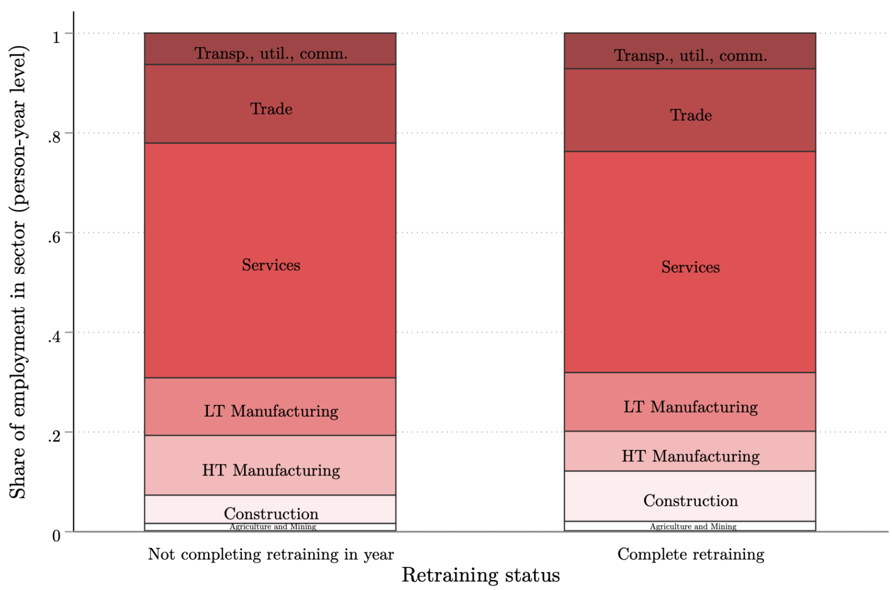
Source: SIAB.

Figure A.4: RETRAINING BY EDUCATION



Source: SIAB.

Figure A.5: RETRAINING BY SECTOR



Source: SIAB, among workers employed in year of retraining.

Finally, we consider the labor market status of workers immediately before and after retraining. We classify workers as prior employed if they are employed within the thirty-

Table A1: Transitions before and after retraining
Share of workers

		To Unemployment	Employment
From	Unemployment	53	18
	Employment	7	22

Source: SIAB. $N = 717,298$ retraining episodes

Table A2: Retraining transitions, employed workers

			New firm	
	Unemployment	Previous firm	Previous industry	New industry
Share	24	50	4	22

Source: SIAB. $N = 204,670$ retraining episodes. Pre- and post-retraining status based on thirty-day window before the start and following the end of retraining episode, respectively.

day window preceding the start date of the retraining program. Similarly, workers are post employed if they are employed within the thirty-day window following the end date of retraining. Table A1 displays the transition matrix. Seventy one percent of retraining participants are not employed in the thirty days leading up to the start date of the retraining program. Of those, three quarters (=53/71) do not find employment right away after they finish, but instead continue searching.

Table A2 displays more detail regarding the bottom row of Table A1. Of the workers who are employed prior to retraining, a quarter of them become unemployed. About half transition stay at their firm, while another quarter move to a new industry. (Industries are defined at the three digit NACE level, so they are more granular than the aggregate sectors described in Figure A.5). Conditional on changing firms, very few workers return to the same industry.

We summarize the relationships between retraining and labor market status in the following pair of facts.

Fact 3. *Most retraining is out of unemployment.*

Fact 4. *Among workers who are employed prior to retraining, about half return to their firm afterward; the remainder move to unemployment or a new industry.*

B Proofs

B.1 Retraining hazard

We prove the inequality in (13).

Proof. Write the variance of $\tau^i \lambda^i(j)$ as $\mathbb{V}(\tau^i \lambda^i(j))$, defined by

$$\mathbb{V}(\tau^i \lambda^i(j)) = \sum_i \left(\tau^i \lambda^i(j)\right)^2 - \left(\sum_i \tau^i \lambda^i(j)\right)^2 \geq 0.$$

To ensure that (13) is non-positive, it suffices to show that $\sum_i \tau^i \lambda^i(j) \left(\tau^i \lambda^i(j) - \tau^i \overline{\lambda}(j)\right)$ is weakly larger than $\mathbb{V}$, hence non-negative. This in turn is implied by

$$\sum_i \left(\tau^i\right)^2 \lambda^i(j) \overline{\lambda}(j) \leq \left(\sum_i \tau^i \lambda^i(j)\right)^2.$$

Using the definition of $\overline{\lambda}(j) = \sum_i \tau^i \lambda^i(j)$, the previous inequality reduces to

$$\sum_i \left(\tau^i\right)^2 \lambda^i(j) \leq \sum_i \tau^i \lambda^i(j),$$

which is immediate since $\tau^i \leq 1 \;\; \forall i$. □

C Calibration

C.1 Expectation Maximization with Dynamic Types

The algorithm draws on Arcidiacono and Miller 2011, with some minor modifications on account of retraining. For a gentle introduction to EM, chapter 8 of Friedman, Hastie, and Tibshirani 2008 is a helpful resource.

Algorithm Start with an initial guess $q_n^{i,(0)}(s)$, $\Theta^{(0)}$. Then, at the m-th iteration:

1. Compute the empirical probability of choosing to retrain, conditional on being type i. Call this $\hat{\delta}^{i,(m)}$. It is equal to

$$\hat{\delta}^{i,(m)} \equiv \frac{\sum_n \sum_t \rho_{nt} u_{nt} q_n^{i,(m)}(s_{nt})}{\sum_n \sum_t u_{nt} q_n^{i,(m)}(s_{nt})}. \tag{17}$$

This is a bin estimator, equal to the type-probability-weighted mass of retrainers divided by the type-probability-weighted mass of unemployed.

2. Evaluate the likelihood $l_n^{i,(m)}(s, \Theta^{(m)}; D)$ (expression given below).

3. Construct the empirical type transition matrix $\hat{\omega}^{i\ell}$, with typical entry

$$\hat{\omega}^{i\ell,(m)} \equiv \frac{\sum_n \sum_t \rho_{n,t-1} q_n^{i,(m)}(s_{nt-1}) q_n^{i,(m)}(s_{nt})}{\sum_n \sum_t \rho_{n,t-1} q_n^{\ell,(m)}(s_{nt-1})}. \tag{18}$$

This is a bin estimator for the share of retraining episodes bridging type i spells and ℓ spells.

4. Update $q^{i,(m+1)}$ by Bayes' rule according to

$$q_n^{i,(m+1)}(s) = \frac{\overline{q}^{i,(m)} l_n^{i,(m)}(s, \Theta^{(m)}; D)}{\sum_\ell \overline{q}^{\ell,(m)} l_n^{\ell,(m)}(s, \Theta^{(m);D})}. \tag{19}$$

$\overline{q}^{i,(m)}$ is the prior probability for q, which is given by cases:

- For those who do not retrain, the prior distribution of worker i's type at the m-th iteration is the population average,

$$\overline{q}^{i,(m)} = \tau^{i,(m)} = \frac{1}{N} \sum_n \frac{1}{S(n)} \sum_s q_n^{i,(m)}(s).$$

- For those who do retrain, the prior distribution is worker-specific and given by the dot product of the previous type probability and the corresponding column of the type transition matrix:

$$\overline{q}_n^{i,(m)} = \sum_{\ell'} q_n^{\ell',(m)}(s-1) \hat{\omega}^{\ell' i,(m)}.$$

5. Compute $\Theta^{(m+1)} = \arg\max_\Theta \log \mathscr{L}\left(\Theta, q^{i,(m+1)}; D\right)$.

6. Return to step 1 and repeat until convergence.

7. After the parameters have converged, compute the retraining failure rate as the relevant element of the bin estimator 18

$$\hat{\nu} = \frac{1}{I} \sum_i \hat{\omega}^{ii}$$

Since we assume that individuals *choosing* to remain the same type are not observed to be retraining, the individual-type-probability-weighted likelihood of remaining the same type for those observed retraining corresponds to the likelihood of retraining failure.

8. Lastly, recover the empirical estimate of the structural transition matrix $\hat{\pi}^{i\ell}$ by

$$\hat{\pi}^{i\ell} = \begin{cases} 1 - \frac{\hat{\delta}^i}{\varrho} & \text{if } i = \ell \\ \frac{\hat{\delta}^i}{\varrho} \frac{\hat{\omega}^{i\ell}}{1-\hat{\nu}} & \text{if } i \neq \ell. \end{cases}$$

The first case comes from the fact that the estimated retraining rate is equal to the arrival rate ϱ multiplied by the retraining probability $1 - \pi^{ii}$ (i.e., the probability that a worker picks anything save his own type). To interpret the second case, the empirical transition probability $\omega^{i\ell}$ from (18) is equal to the probability of choosing to retrain and being successful, $\pi^{i\ell}(1 - \nu)$, divided by the pool of potential retrainers, $1 - \pi^{ii}$; algebraic rearrangement yields the result.

Likelihood Let $\widetilde{\Theta}$ be the parameter guess. The log-likelihood equals the individual-type-probability weighted sum of the likelihood contributions of each individual n at each time t, were the individual to be of type i:

$$\log \mathscr{L}\left(\widetilde{\Theta}, q^i_{n(s);D}\right) = \sum_n \sum_i \sum_t q^i_n(s_{nt}) \log l^i_{nt}(\widetilde{\Theta}; D) \tag{20}$$

In turn, the individual-by-time-by type-specific likelihood contributions are given by

$$l^i_{nt} = \begin{cases} \hat{\delta}^i \left[u_{n,t-1} \left(1 - \sum_j \widetilde{\lambda^i(j)}^0 \right) + (1 - u_{n,t-1})\, \mu \right] \\ \qquad \text{if } u_{nt} = 1 \text{ and } \rho_{nt} = 1 \\ \\ \left(1 - \hat{\delta}^i\right) \left[u_{n,t-1} \left(1 - \sum_j \widetilde{\lambda^i(j)}^0 \right) + (1 - u_{n,t-1})\, \mu \right] \\ \qquad \text{if } u_{nt} = 1 \text{ and } \rho_{nt} = 0 \\ \\ f\left(\frac{\log \omega_{nt} - \sum_j d_{njt} \log\left(\widetilde{\varepsilon^i(j)}^0 \right)}{\varsigma} \right) \left[u_{n,t-1} \sum_j d_{njt} \widetilde{\lambda^i(j)}^0 + (1 - u_{n,t-1})\,(1 - \mu) \right] \\ \qquad \text{if } u_{nt} = 0 \end{cases} \tag{21}$$

where $f(\cdot)$ denotes the standard normal density.

C.2 Estimated parameters

Table A3: Job offer arrival rate estimates, $\lambda^i(j)$

	Worker type (i)		
Sector (j)	1	2	3
Agriculture and Mining	0.0129	0.0032	0.0062
Manufacturing			
food and beverage	0.0127	0.0031	0.0062
clothes	0.0124	0.0003	0.0051
paper and related	0.0129	0.001	0.0057
fuel and chemicals	0.0125	0.0011	0.0051
mineral products	0.0127	0.0022	0.0055
metals	0.0126	0.0024	0.0060
industrial machinery	0.0128	0.0026	0.0065
electronics	0.0127	0.0024	0.0062
auto	0.0127	0.0013	0.0053
other transport	0.0124	0.0002	0.0049
misc.	0.0124	0.0007	0.0052
Other			
Construction	0.0168	0.0044	0.0071
Trade	0.0147	0.0116	0.0126
Transportation, utilities, and communication	0.0138	0.0067	0.0082
Services			
skilled	0.0136	0.0067	0.0090
ed/med	0.0172	0.0124	0.0089
other	0.0184	0.0164	0.0223

Source:

Table A4: Value added per worker estimates, $\varepsilon^i(j)$

	Worker type (i)		
Sector (j)	1	2	3
Agriculture and Mining	0.6484	0.6740	0.5175
Manufacturing			
food and beverage	0.6835	0.8305	0.6436
clothes	1.0007	0.9839	0.9241
paper and related	1.0848	0.9285	0.6077
fuel and chemicals	0.9525	1.5198	0.8838
mineral products	0.5976	1.0885	0.7703
metals	0.9011	1.0933	0.7982
industrial machinery	1.1582	1.2783	0.9159
electronics	1.1396	1.2768	0.8643
auto	0.9445	1.6248	1.2812
other transport	0.7752	1.4229	1.2111
misc.	0.9029	0.9333	0.7592
Other			
Construction	0.9094	0.6235	0.6757
Trade	0.7145	0.8789	0.5418
Transportation, utilities, and communication	0.6197	0.8978	0.5196
Services			
skilled	0.7799	1.1226	0.6083
ed/med	0.3967	0.9915	0.6399
other	0.6185	0.4748	0.8653

Source: